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Investment-Risk Protection Rider for Unit-Linked Insurance

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Abstract

This paper designs an investment-risk protection rider that provides annual return bounds (a floor and a cap) at no explicit cost to the policyholder. The study models two scenarios using the Indonesia Stock Exchange Composite Index (IHSG) as the underlying asset: the first assumes the availability of derivative options to form a zero-cost collar, while the second assumes no options are available, forcing the company to use delta-neutral dynamic hedging. A primary novelty of this research is the demonstration of theoretical option pricing on non-normal return assets and the formulation of a "Semi Non-homogenous Double-Exponential Jump Diffusion" (SNDEJD) model, which is developed to resolve an analytical calculation issue found in the original Non-homogenous Double-Exponential Jump Diffusion (NDEJD) model's pricing formula, thereby allowing for theoretical option pricing while capturing long-term parameter shifts. The study concludes that if options are available, the rider is highly viable, offering a 0% return floor and a median cap of 14.9% with no risk to the insurer. However, the delta-neutral dynamic hedging approach is found to be ineffective and risky, as the Black-Scholes hedging model fails to cover the jump risks in the non-normal IHSG returns, leaving the company exposed to significant losses unless a much lower cap is set.

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1. INTRODUCTION

During the COVID-19 period, more than three million investment-linked insurance (*Produk Asuransi Yang Dikaitkan dengan Investasi*/PAYDI) policyholders surrendered their policies.[1] One of the factors is the declining investment values during COVID-19 which directly affected their unit-linked account. The significant drawdown in the equity market is consistent with broader studies on the volatility of emerging markets during the pandemic, which showed deeper stress compared to developed economies [1]. This highlights the need for product designs that can protect investment outcomes during market turbulence while preserving an expected return above the risk free rate. In several foreign markets, insurers have launched structured investment products that incorporate return floors and caps mechanisms for the investment account's annual return.[2] These mechanisms are conceptually similar to Variable Annuities with investment guarantees, which have been extensively studied and valued in actuarial literature [2]. The proposed investment mechanism can safely be applied using derivative instruments. However, the derivative market in Indonesia remains relatively inactive. Although *structured warrants* are available on the Indonesia Stock Exchange, the trading volumes are still limited.[3] One possible way is to cooperate with a financial institution that has a license to issue structured warrants. However, this requires a sufficiently capable option pricing model for Indonesian stocks, which do not have a normal distribution [3]. The second option is not to buy options, but to perform

[1] Law Justice.co. Source: https://law-justice.co/artikel/116762/jutaan-nasabah-asuransi-ramai-ramai-tutup-unit-link-ini-sebabnya/
[2] Pruco Life Insurance Company. Source: https://prudential.scene7.com/is/content/prudential/1005990_FoundersPlusConsumerBrochure
[3] Indonesian Stock Exchange. Source: https://www.idx.co.id/id/data-pasar/structured-warrant-sw/informasi-structured-warrant/

dynamic hedging to cover the option's payoff. However, this will still leave residual risk for the insurance company.

This paper will construct a unit-linked investment protection rider by giving lower and upper bound to the annual return without extra explicit cost to the policy holder. The rider will be constructed under two contrasting scenarios: one assuming availability of derivative options to define lower and upper return limits, and the other assuming no options are available where the company will apply delta neutral dynamic hedging. For the first approach, due to limited hitorical option prices, this study will build on earlier work introducing the Non-homogeneous Double-Exponential Jump Diffusion (NDEJD) model for stock return dynamics to price the option theoretically [4]. For the second approach, a previous study have demonstrated that when only the underlying asset is available for hedging in a jump-diffusion environment (which is the case for this scenario), the only feasible strategy is delta-neutral hedging [5]. Based on the background above, the specific objectives of this research are threefold: to develop the Semi Non-homogenous Double-Exponential Jump Diffusion (SNDEJD) model to resolve analytical limitations in pricing options for non-normal assets like the IHSG, to determine the feasible annual return upper bound (cap) that allows for a zero-cost protection rider under an option-available scenario using a collar strategy, and to evaluate the effectiveness and risk exposure of a delta-neutral dynamic hedging strategy in a scenario where derivative options are unavailable.

2. METHODS

This study designs a unit-linked insurance structure that limits annual portfolio returns within predetermined bounds using stochastic simulation and option-pricing theory on an qeuity account unit-linked product defined by OJK guidelines [6]. The portfolio weight allocation on the equity and fixed income component of the unit-linked account may be determined by the insurance company with a minimum of 80% equity allocation [6]. However, the proposed rider is designed to attach only to the equity component of the policyholder's unit-linked investment account. The Indonesia Stock Exchange Composite Index (IHSG) is used to represent the domestic equity market due to its broad sectoral coverage and long historical record, making it a reliable benchmark for Indonesian equity performance. Historical IHSG prices from January 1990-July 2025 are obtained from Investing.com [7]. The methodology will be split into two parts based on the two scenarios: the development of NDEJD model to price an at-the-money put option and finding a strike price at which a call option has the same premium as the put option, and developing a delta neutral dynamic hedging under the theorithical NDEJD return distribution of the IHSG. This study will then compare the results and the risks that the insurance company will bear upon launching this rider.

The outcome of this study will be the reasonable upper bound that a company can offer depending to their risk apetite and required return. Due to the long-term natur of unit-linked products, the upper-bound projection will be done untill 30 years dependent to different market situations in the future: very high (95^{th} percentile of market return), high (80^{th} percentile), normal (50^{th} percentile), low (20^{th} percentile), and very low (5^{th} percentile). The upper bound offer will be renewed every year and policy holders can choose wether to take or not the protection. Different upper bounds calculated every year and every market situations are to approximate the upper bound that the company will offer.

2.1 Theoretical NDEJD Pricing to Create a Zero-Cost Collar Approach

The NDEJD model assumes the log-return of stock prices follows a jump diffusion model. This modeling framework builds upon the classical Jump Diffusion model by Merton [8] and the Double Exponential Jump Diffusion (DEJD) model by Kou [9], defined as follows:

$$\frac{dS_t}{dt} = \mu\, dt + \sigma\, dW_t + dJ_t, \tag{1}$$

where *t* represents the time, S_t represents the stock price at time t, μ serves as the drift component, σ represents the non jump log-return's volatility, W_t is a standard Wiener process, and J_t represents the jump at time *t* [4]. To obtain the parameters of the model, this study will first obtain the historical daily prices of IHSG and categorize each daily log-return as a jump component or a diffusion component. This study will find an optimal benchmark to categorize the return. The optimal benchmark is the benchmark that will suffice the NDEJD model's assumption: Normal distribution upon the non-jump diffusion process, Poisson Process upon the yearly jump frequency, and non-homogenous double exponential (NDE) distribution on the jumps severity. Considering the difficulty on testing assumption sufficiency on the NDE distribution assumption, this study will only optimize the Normal and Poisson Process assumption to find the benchmark. The benchmark will be in terms of $\mu \pm k\sigma^{(0)}$, where μ is the mean of the whole daily log-return and $\sigma^{(0)}$ is the standard deviation (jumps included). Every daily log-return outside the interval $[\mu - k\sigma^{(0)}, \mu + k\sigma^{(0)}]$ will be considered a jump (optimal $k \in \mathbb{R}$ will be found to determine the benchmark).

After deciding on the benchmark, we will first extract the constant drift component (μ). The non-jump component (diffusion component) will be fitted to a Normal distribution $(0, \sigma)$, while the annual jump frequency will be fitted to a Poisson distribution. However, a problem arises when fitting the jumps to the NDE distribution: NDE distribution's density is highest at 0, while the jumps after the drift is extracted will be either more than $k\sigma^{(0)}$ or less than $-k\sigma^{(0)}$ (meaning that the density at 0 shoule be zero). To overcome this problem, we will use the memoryless property of Exponential distribution and the fact that compound Binomial-Poisson is a Poisson distribution. NDE distribusion is a Double Exponential (DE) distribution with revolving exponential parameters. If we forcibly fit the jumps to a DE with decaying rates η for positive jumps and $\tilde{\eta}$ for negative jumps, the model has a probability of $\check{p} = (p\left(1 - e^{-\eta k\sigma^{(0)}}\right) + (1-p)(1 - e^{-\tilde{\eta} k\sigma^{(0)}}))$ to yield a non-jump return, where p is the probability that a jump is a positive jump. This means that given there are N jumps, the conditional real jumps will follow a Binomial distribution (N, $1 - \check{p}$). Considering that we want the unconditional real jumps to follow a Poisson distribution, we can define N to follow a Poisson distribution so that the unconditional real jumps follow a compound Binomial Poisson which is a Poisson distribution [10].

Upon this observation, the Poisson parameter of the NDEJD model will be the Poisson (λ) under the compound Binomial Poisson distribution, which using the method of moments can be fitted as

$$\lambda = \frac{average\ yearly\ jump\ count/252}{1 - \check{p}} \tag{2}$$

where 252 is the assumed count of working days where stocks can be traded. Even though the jumps severity model can now yield returns inside the benchmark, the density of the jumps outside the benchmark should be proportional to the original empirical distribution of the jumps. The parameter can be obtained by assuming that the data we have is truncated along the benchmark interval, meaning that we will only use the jumps. To achieve this, we can use the memoryless property of exponential distribution which also prevails on DE and NDE distribution and fit the benchmark-excess version: $(actual\ jump - k\sigma^{(0)})$ for positive jumps and $(actual\ jump + k\sigma^{(0)})$ for negative jumps. Under these circumstances, the model will analytically obtain the frequency parameter and obtain the severity parameter within only one assumption: the non-jump components yielded from the jump processes does not affect the normal distribution of the diffusion component.

The evolution of the jumps severity parameter will be obtained by fitting positive and negative jumps seperately. The exponential rate of the k^{th} positive or negative jump will be obtained by fitting the first untill the k^{th} positive or negative jump using method of moments to obtain η_k as the exponential rate of the k^{th} positive jumps and $\check{\eta}_k$ for the negative jumps. After obtaining positive and negative jumps' exponential rate evolution, the rate will be forecasted individually using time series model. It is important to note that under long-term projections, the evolution of the NDE parameter can shift the drift and jump frequency. However, the NDEJD's option pricing model assumes a constant drift parameter and frequency. To address this issue, the model's drift and jump frequency will be a piecewise constant function over every year, which will be calculated at every begining of the year as $\lambda_t = \frac{average\ yearly\ jump\ count/252}{1 - \check{p}_t}$, where $\check{p}_t = (p(1 - e^{-\eta_t k\sigma}) + (1-p)(1 -$

$e^{-\tilde{\eta}_t k\sigma}$)) and $\mu_t = \mu - \lambda_t(\frac{p}{\eta_t} - \frac{1-p}{\tilde{\eta}_t})$. The value of λ_t will replace the previous poisson parameter expressed in Equation (2), and μ_t will replace the value of μ.

After obtaining the NDEJD parameters, the study will use a Monte Carlo simulation to obtain the annual stock prices on the 5 scenarios (based on the 95th, 80th, 50th, 20th, and 5th percentiles). On each year and each percentile, the study will calculate the at-the-money put option (at strike price $K_{t,\alpha}^{d} = S_{t-1,\alpha}$) so that the minimum stock price we obtain on the α^{th} percentile scaneario at time *t* is $S_{t-1,\alpha}$ and obtain 0% annual return lower bound. However, the put option requires a premium to pay. To make the protection zero-cost, the portfolio will take a short position on a call option with a strike price ($K_{t,\alpha}^{u}$). This study will assume a bid ask spread of 5%. Under this assumption, the strike price of the call option can be calculated by finding $K_{t,\alpha}^{u}$to solve the equation:

$$C_{t,\alpha}(K_{t,\alpha}^{u}, 1)(1 + 2.5\%) = P_{t,\alpha}(S_{t-1,\alpha}, 1)(1 - 2.5\%), \tag{3}$$

where $C_{t,\alpha}(K, T)$ is the premium of a call option at time *t* under α^{th} percentile scaneario with strike price *K* and time to maturity T, and $P_{t,\alpha}(K, T)$ serves for the put option. The analytical formula to calculate put and call option prices are derived by the previous study [4]. By doing so, the maximum price we can obtain at time *t* is $K_{t,\alpha}^{u}$ and the annual return upper bound can be defined as $CAP_{t,\alpha} = K_{t,\alpha}^{u}/S_{t-1,\alpha}$.

2.2 The Proposed Semi Non-Homogenous DEJD Model (SNDEJD) Model

While the NDEJD model captures the time-varying nature of jump parameters, its analytical option pricing formula derived by Lin et al. [4] encounters a singularity issue when the exponential rate parameters of different time steps coincide (η_i=η_j). As shown in the projection results (discussed later in Section 3), the projected negative jump rates tend to stabilize, causing the denominator in the pricing kernel $\prod_{i=1,i\neq j}^{n} \frac{\eta_j}{\eta_j - \eta_i}$ to approach zero.

To resolve this, this study develops the Semi Non-Homogenous Double-Exponential Jump Diffusion (SNDEJD) model. Unlike the fully continuous NDEJD, the SNDEJD model imposes a piecewise constant assumption on the model parameters over the option's maturity period. Specifically, for an option with maturity *T* (where *T*=1 year in this study) priced at time *t*, the parameters $\theta_t = \{\mu, \sigma, \lambda, p, \eta, \tilde{\eta}\}$ are assumed constant within the interval [*t*, *t* +*T*]. Formally, the dynamics of the asset price under SNDEJD follow the standard DEJD, but the parameters μ_t, σ_t, η_t, $\tilde{\eta}_t$ are updated annually based on the long-term projection but remain fixed during the pricing of the 1-year option. This assumption allows the use of the closed-form analytical solution for option pricing derived by Kou [9], effectively bypassing the singularity issue in Lin et al.'s formula while still capturing the long-term non-homogeneity of the market through annual parameter updates.

2.3 Delta-Neutral Dynamic Hedging Approach

The dynamic hedging approach's main idea is to create a dynamic portofolio that can replicate the pay-off of the insurance company upon selling the rider which is equivalent of replicating a portfolio with a long at-the-money put and short call at a higher strike price. This approach is based on the limitation that the insrance company cannot find suitable options because the option market in Indonesia remains inactive. Under this circumstances, the only asset that can be used to hedge is the underlying asset itself, leaving the delta-neutral dynamic hedging approach as the only way to hedge this position [5]. The dynamic hedging account will be created with no extra cost on two assets whoose weight allocation will be updated on every working days (252 times a year): the underlying stock (IHSG) and lending or borrowing money at risk free rate. Short possitions on IHSG can be obtained by borrowing from the main policy holder's unit linked account and can return the stocks upon recovering the short possition.

A previous study has estimated that a liquid stock in Indonesia will have a bid-ask spread around 0.06%. Therefore this study will asume that the dynamic hedging is performed under 0.06% stocks bid-ask spread. To calculate the underlying asset's exposure each day, we can use the delta of long put and short call obtainable from the Black Scholes model [11]:

$$\Delta^c = e^{-qT}\phi(d_1), \tag{4}$$

$$\Delta^p = e^{-qT}(\phi(d_1) - 1), \tag{5}$$

where Δ^c and Δ^p are respectively the delta of call and put options, q is the dividend rate, T is the time to maturity, and $\phi(d_1)$ is the cumulative standard normal probability with $d_1 = \frac{\ln\left(\frac{S}{K}\right)+(r-q+0.5\sigma^2)T}{\sigma\sqrt{T}}$, where S is the stock price, K is the strike price, σ is the stock's volatility, and r is the risk free rate.

The unit of IHSG that a dynamic hedging account will take for every IHSG unit of the main unit-linked account can be calculated by $\Delta_{t,S_t} = \Delta^p_{t,S_t} - \Delta^c_{t,S_t}$ to hedge the short put and long call position, where Δ^p_{t,S_t} and Δ^c_{t,S_t} are the delta of put and call option that are calculated at time t given that the stock price at time t is S_t, calculated using Equation (4) and Equation (5). Positive Δ_{t,S_t} means a long position on IHSG which will be bought on the ask price and borrowing the required money at risk free rate, while negative Δ_{t,S_t} means a short position on IHSG which will be sold on the bid price and lending money at risk free rate. The money on risk free rate will be compounded continuously on 5.8% interest rate, which is the average of BI rate from 2009-2025, which is the longest time-frame available on *Badan Pusat Statistik* (BPS) [12].

The desired result is for the dynamic hedging account, which is constructed with no cost, to have a pay-off replicating the pay-off that the company must pay. The effectivity of the dynamic-hedging method will be quantified using a Monte Carlo simulation by utilizing the stochastic NDEJD's IHSG return. The quantification will compare the standard deviation of company's pay-off with and without dynamic hedging, as well as the percentiles of the pay-off. The risk-premium that the company can obtain is the expected return of the dynamic hedging portofolio, which will be tested on several upper bound offering so that the insurance company can choose the upper bound to offer that matches their risk-premium requirement.

3. RESULT AND DISCUSSION

3.1 NDEJD Model's Parameters

By applying the steps provided in subsection 2.1, the parameters of the model were obtained. The optimal benchmark was found through a grid search of values of k that maximized the p-value of the Kolmogorov-Smirnov test for the diffusion component's normality. It maximized the p-value of the Chi-Square test for the yearly jump frequency's Poisson distribution. The test results on finding the optimal k are presented in Figure 1. As seen in the figure, $k = 1.4$ was optimal for both the diffusion's normality assumption and the yearly jump frequency's Poisson assumption. Therefore, every daily log-return of IHSG outside $[\mu - 1.4\sigma^0,\ \mu + 1.4\sigma^0]$ was considered a jump, where $\mu = 0.028\%$ and $\sigma^0 = 1.37\%$, meaning that the lower bound was at -1.38% and the upper bound was at 1.95%. As a result of this benchmark, the average annual jump frequency was 26.43 jumps, meaning that 10.5% of the daily returns were considered as jumps and the probability of positive jumps as $p = 0.466$.

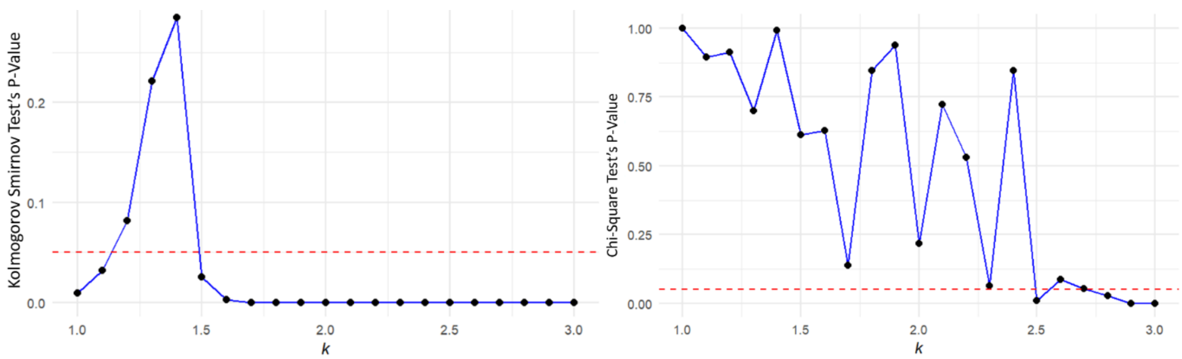

Figure 1. Assumption tests results through values of *k*.

The evolution of the positive and negative jumps' exponential rate, as well as their projection, is presented in Figure 2, where the dotted vertical line marks the start point of the projection. For positive jumps, the exponential rate followed the ARIMA(4, 2, 2) model, and the negative jumps followed the ARIMA(3, 1, 2) model. The exponential rate of positive jumps was projected to increase, while the projection of negative jumps was almost constant. This means that the severity of positive jumps tends to decline because a higher exponential rate yields a lower expected value. In contrast, the severity of negative jumps tends to stay the same. After obtaining the values of η_k and $\tilde{\eta}_k$, the values of λ_t and μ_t were calculated with the asumption that for every increment in *t* (year), the index of jumps (k) increased by 12 for positive jumps and 14 for negative jumps (based on the average annual positive and negative jumps frequency). The expected value of an exponential distribution with rate η is $\frac{1}{\eta}$. The projection of the positive jumps takes a value of around 80-120, meaning that the expected magnitude of the positive jumps is at around 0.833% to 1.250%. For negative jumps, the projection takes a value of around 78, meaning that the expected magnitude of negative jumps is at around -1.282%.

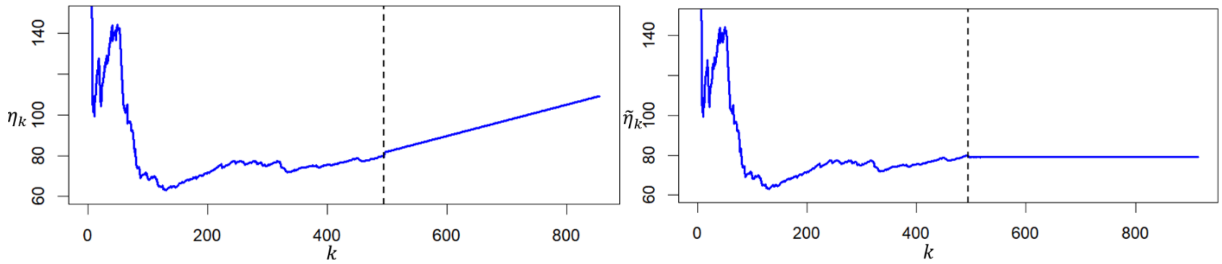

Figure 2. NDEJD Positive and negative jumps' exponential rate projection.

Other than the non-homogeneous exponential rates, other parameters of the NDEJD model are presented in Table 2. Parameter μ has an estimated value of 0.028%, meaning that under the assumption of 252 working days, the expected annual return of IHSG is 7.056%. However, this value of μ isn't the parameter that will be used on the NDEJD model, since the model will use μ_t whose value will fluctuate to compensate for the fluctuation of the non-homogeneous double exponential rates as explained in Section 2.1. The parameter σ represents the standard deviation of the daily non-jump returns. The value of σ is expected to be less than σ^0 since σ^0 includes jump returns in its calculation. The value of λ is at 26.3, meaning that the expected count of jumps within a year is at 26.3. The benchmark of the jumps is defined by $[\mu - 1.4\sigma^0,\ \mu + 1.4\sigma^0]$, meaning that the lower bound is at -1.38% and the upper bound is at 1.95%. The parameter *p* represents the probability that a jump is a positive jump, meaning that negative jumps appear more frequently in the historical data, reflected by the value of *p* being less than 0.5. Finally, the value of *r* represents the annualized risk-free rate, which is obtained from historical BI rate data.

Table 2. NDEJD model's parameter summary.

Parameter	Estimated Value
μ	0.028%
σ	0.741%
σ^0	1.371%
λ	26.30
p	0.46
r	5.63%

3.2 Option Pricing Results and Upper Bound/Cap Calculation

After obtaining the parameters of the NDEJD model, IHSG's stock prices were projected using a Monte Carlo simulation under 5 scenarios (based on the 95th, 80th, 50th, 20th, and 5th percentiles). After obtaining the stock prices projection, within every year of projection and every scenario, this study calculated the at-the-money put option using the analytical NDEJD formula for put and call options derived by the previous study. A problem arised upon calculating the option prices, since the analytical formula to calculate the option contained a component of $\prod_{i=1,i\neq j}^{n} \frac{\eta_j}{\eta_j-\eta_i}$ which caused singularity issue due to negative jumps with parameters presented on Figure 2. Therefore, the option prices were calculated using the proposed SNDEJD model explained in Section 2.2.

To obtain the new exponential rate evolution, the values of η_t and $\tilde{\eta}_t$ were calculated respectively as the average of 12 positive parameter projections and the average of 14 negative parameter projections to avoid biased estimation of the parameter. The new exponential rates projection under the SNDEJD model is presented in Figure 3. As seen in Figure 3, for the experimental data, the SNDEJD approach still captures the long-term jump severity trend while allowing the usage of the DEJD model for option prices. Therefore, the SNDEJD approach was used for option pricing, while every other calculation, including stock projections, still applied the NDEJD model.

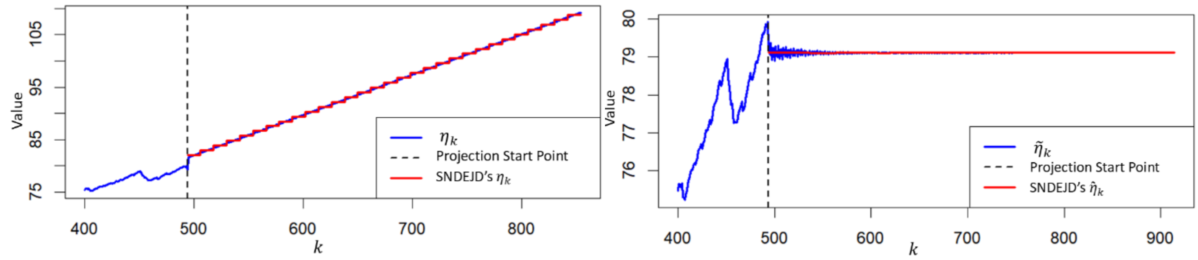

Figure 3. SNDEJD positive and negative jumps' exponential rate projection.

After obtaining the NDEJD parameters, we calculated the at-the-money put option premium, so as the strike price of the call option to offset the put option's premium. Afterwards, we proceeded with calculating the protection rider's cap/upper bound. The result of the cap projection on different scenarios is presented in Figure 4. The median of the cap is 14.9%. The variation of the cap offering in different scenarios is due to the accumulated average return obtained within each scenario. A good scenario (high quantile scenario) was shown to have a larger accumulated return, meaning that the value of μ at the starting year of option pricing was higher than those in the worse scenario. The increment of μ reduced the premium of the at-the-money put option, since higher μ means that the stock has a higher tendency to increase in price. The lower the premium of the put option, the higher the strike price of the call option, because higher strike price call options are cheaper than ones with a lower strike price. Thus, the cap in high scenarios was higher than the cap in low scenarios.

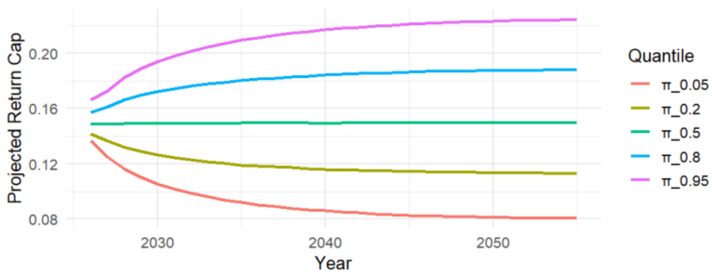

Figure 4. Projected investment protection rider's cap using theoretical option pricing.

3.3 Dynamic Hedging Results

Firstly, the dynamic hedging was done on a scenario where the cap offered by the insurance company is the median of the cap in 2026, which is 14.9%. The effectiveness of dynamic hedging is presented by comparing the company's loss density on providing the investment protection rider without dynamic hedging and with dynamic hedging, in Figure 5. As presented in Figure 5, the company's pay-off density is higher at

0, and lower on the outliers. This observation means that by applying dynamic hedging on top of the investment protection rider, the company has decreased its risk exposure.

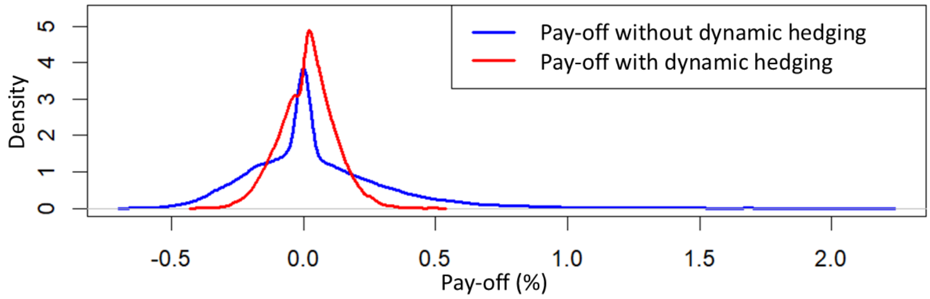

Figure 5. Dynamic hedging's effectivity comparison.

However, it is important to notice that under this scenario, the company is still exposed to some degree of risk. Compared to the first scenario, where options are available, there is a risk that the company loses money, whereas the other scenario has no risk at all. To accept this risk, it is natural that the company demands a risk-premium. The risk premium is in the form of a cost to obtain the investment protection, or by setting a cap so that the expected return for the company is positive. In this scenario, the expected return for the company is -0.05%, mostly due to the 0.06% bid-ask spread on the daily transaction of the stock. This means that it is unreasonable for the company to offer this investment protection. While one way to solve this problem is by explicitly setting a cost to obtain the protection rider, another way is to set a lower cap for the policyholder. The risk that the company takes, as well as the risk premium on several levels of cap, is presented in Table 2. As seen in Table 2, the company can expect a profit of 1.5% of the portfolio's value upon setting a 10% cap. However, the company is still exposed to losses of more than 20%. The ineffectiveness of the dynamic hedging lies on the assumption that the stock return follows a normal distribution, which is clearly not the case for IHSG. This inefficiency aligns with established literature stating that discrete hedging in the presence of transaction costs [13] and jump discontinuities [14] makes perfect risk replication impossible. This decreases the delta neutral hedging's effectiveness to cover the pay-off.

Table 2. The company's risk and risk-premium on different levels of cap.

u	E[r]	$\pi_{0.01}$	$\pi_{0.05}$	$\pi_{0.2}$	$\pi_{0.8}$	$\pi_{0.95}$	$\pi_{0.99}$	σ
14%	0.2%	-27.4%	-18.9%	-9.4%	9.6%	18.6%	26.9%	0.117
13%	0.5%	-26.7%	-18.4%	-9.2%	9.6%	18.8%	27.4%	0.116
12%	0.8%	-25.3%	-17,6%	-8.4%	9.7%	19.0%	28.0%	0.112
11%	1.1%	-25.1%	-16.8%	-7.8%	9.8%	19.2%	28.4%	0.109
10%	1.5%	-23.6%	-16.1%	-7.0%	9.9%	19.4%	28.8%	0.104

3.4 Regulatory Context and Practical Implications

The feasibility of the proposed investment-risk protection rider must be viewed within Indonesia's PAYDI regulatory framework. Insurers must ensure that prudent risk management, transparent cost structures, and adequate capital support any investment feature or embedded guarantee. These requirements emphasize that structured features must be hedged effectively and not expose the insurer to undue solvency risk.

In this context, the option-based zero-cost collar provides a clearer compliance pathway, since its payoff can be fully replicated when derivatives are available. This aligns with OJK's emphasis on demonstrable hedging and controlled risk exposure. However, the practical use of this approach is limited by the low liquidity of structured warrants and other derivative instruments in Indonesia, which may not support consistent or large-scale hedging.

If derivatives are unavailable, insurers may use delta-neutral dynamic hedging, but our results show that this approach leaves residual risk due to jump-diffusion behavior and transaction costs in the IHSG. From a regulatory standpoint, this residual risk must be explicitly incorporated into the product's risk-management documentation, capital assessment, and policyholder disclosures. Lower caps or explicit fees may be required to keep expected returns positive while maintaining compliance.

Operationally, insurers considering this rider must choose between partnering with licensed issuers of structured warrants to support the collar strategy or offering a dynamically hedged version adjusted to their risk appetite. Overall, the development of Indonesia's derivative market would substantially improve the viability of innovative PAYDI product designs.

4. CONCLUSIONS

This study develops the Semi Non-Homogeneous Double-Exponential Jump Diffusion (SNDEJD) model to address the singularity issue in the NDEJD option-pricing formula while retaining the ability to capture long-term shifts in jump parameters. The SNDEJD approach provides a practical and computationally efficient framework for pricing options on assets with non-normal return characteristics, such as the IHSG.

Based on this model, the option-available scenario demonstrates that an investment protection rider can be offered with a 0% floor and a cap of approximately 14–15% with the median of 14.9% (excluding other expenses in maintaining the rider), without introducing risk to the insurer. However, the feasibility of this approach depends on the future availability and liquidity of derivative instruments in Indonesia.

When options are unavailable, the insurer may rely on delta-neutral dynamic hedging. Although this method reduces loss variability, it leaves material residual risk due to jump behavior and transaction costs, resulting in negative expected returns unless the cap is lowered or an explicit fee is charged. The acceptability of this residual risk ultimately depends on the insurer's risk appetite and capital considerations. If such risk is unacceptable, partnering with institutions capable of issuing put and call options (structured warrants) on the exchange becomes the only practical alternative.

This study does not account for operational costs associated with sourcing options, forming partnerships, or executing dynamic hedging, and it assumes a bid–ask spread that may vary in Indonesia's relatively illiquid market. Future research may consider incorporating other asset classes to diversify the investment account, exploring more advanced hedging techniques, and, once sufficient option data becomes available, applying machine learning models to improve option-price estimation and rider design.

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