IAJ
Indonesian Actuarial Journal
Persatuan Aktuaris Indonesia (PAI)

Vol. 01, No. 02, December (2025)
e-ISSN 3110-6463
https://10.65689/iajvol01no2pp152-163

Cost Component Calculation for Endowment Insurance Premiums on Multiple Life Using the Gompertz Distribution

Ady Febrisutisyanto[1*], Dwi Haryanto[2], Antonia Samantha Damara[3]

[1,2,3] *Actuarial Science, STMA Trisakti, Jakarta, Indonesia*
*Corresponding email: af1802.af@gmail.com

Abstract

This study aims to develop a gross premium model for endowment insurance products under a multiple life setting by incorporating various actual cost components and applying the Gompertz mortality distribution. The proposed model includes acquisition costs at the beginning and end of the first policy year, premium collection costs, and annual policy maintenance costs, all of which are calculated based on present values of benefits and annuities. Parameter estimation is conducted using linear regression with a bounded optimization approach, where all parameters are constrained to be strictly positive to reflect realistic conditions in insurance practice. The simulation results yield parameter estimates of $\alpha_1 = 0{,}97074$, $\alpha_2 = 0{,}912217$, $\beta = Rp\ 19.344{,}46$ *and* $\gamma = Rp\ 20.105{,}23$ *which are considered actuarially reasonable. The high coefficient of determination,* $R^2 = 98{,}46\%$, *indicates that the model has an excellent fit to the gross premium data. This research demonstrates that an actuarial-based cost formulation combined with statistical estimation can serve as an effective and transparent approach in determining premiums for endowment life insurance products with more than one insured.*

Article History:
Received: 7 July 2025
First Revised: 21 July 2025
Accepted: 21 July 2025
Published: 31 December 2025

Keywords:
Gross Premium, Endowment Life Insurance, Multiple Life, Gompertz Distribution, Linear Regression, Bounded Optimization.

1. INTRODUCTION

Death is the danger that all people will eventually encounter. Financial loss may result from this risk, particularly if the family's head of household or primary provider passes away. The existence of a Dual Benefit Life Insurance product, which is a kind of insurance that offers two types of benefits: the benefit if the insured survives until the end of the protection period or dies within the protection period, is one way to reduce this financial loss. One type of dual-benefit life insurance is multiple life insurance Multiple life insurance is a policy that covers two or more lives, with benefits paid out if one of the insured individuals passes away [1]. Whole life multiple life insurance provides a period of coverage as long as all insured individuals are still alive or until at least one insured individual reaches the oldest age. In traditional actuarial literature, multiple life contracts are considered as independent or mutually exclusive events [2].

However, this approach is considered less accurate because, in practice, there is dependence among insured individuals, especially in the family context shows that the assumption of independence can lead to bias in premium determination, making dependency models such as multivariate copulas important in calculating family insurance premiums involving more than two lives. This dependence affects the estimation of the probability of living together and the value of the benefits to be paid [3]. In a life insurance policy, there will certainly be costs that need to be paid by the insured every month, referred to as the insurance premium. The amount of premium that must be paid by the insured is the sum of the pure premium and the loading factor. The pure premium is obtained from the expected occurrence of risk, while the loading factor includes other costs such as administrative fees, operational costs, and so on. The components of endowment life insurance costs consist of initial closing costs, premium collection costs, and maintenance costs [4].

The calculation of pure premiums for dual-purpose multiple life insurance, cost components become an important factor that influences the balance between the obligations of the insurance company and affordability for policyholders. The main cost components in pure premiums include mortality costs, operational costs, and technical reserves that must be met to ensure the sustainability of benefit payments in the future [5]. Inaccuracies in estimating these costs can impact the solvency of the insurance company and the risk balance within the policy portfolio. Additionally, the complexity of multiple life insurance increases uncertainty in calculating the benefit value derived from the combination of survival and death probabilities of more than one insured. Therefore, a model that can better represent risk characteristics is needed so that cost components can be calculated accurately and in accordance with actuarial principles.

In the calculation of cost components in dual-purpose life insurance, mortality tables or distributions that depict mortality rates, such as Weibull, Inverse-Weibull, and Gompertz, are usually used. The Gompertz distribution has become one of the distributions that can be used to predict mortality rates that are more aligned with actual rates. This distribution is widely used for studies on mortality and can reflect the increase in mortality rates and aging with considerable flexibility, accurately and simply. In its application, the Gompertz distribution can be used to estimate the probability of someone surviving and dying, as well as the present value of a lifetime annuity [6].

The Gompertz distribution, which is one of the mortality models for estimating death rates, is often applied in prospective reserve estimation because it can project a more realistic death pattern. This distribution can also accurately and simply depict the increase in mortality risk as age increases. With its high flexibility, Gompertz is often chosen to produce more accurate reserve estimates because it is highly flexible and effective in simulating the processes of aging and mortality. The probability of survival under the Gompertz assumption is lower compared to the survival probability data from the 2011 Indonesian Mortality Table. This is because the Gompertz approach not only estimates the probability of survival but is also sensitive to interest rate changes due to exchange rate fluctuations. In the study, the depreciation of the rupiah against the dollar caused an increase in interest rates, which in turn raised the premium value. The research also indicates that not only the mortality model plays a role in the accuracy of premium calculations, but external economic factors as well [7].

Other research also states that the Gamma-Gompertz mortality law is suitable for modeling the mortality rate of the population. The mortality rate can later be used in the calculation of premiums and annual gross benefit reserves more accurately [8]. Furthermore, the premium for dual-purpose life insurance obtained using the Gompertz law is higher than the premium obtained using the de Moivre law [9]. This is because the de Moivre law assumes a uniform distribution of mortality probability, meaning the probability of death is spread evenly, whereas the Gompertz law describes an exponential increase in the risk of death as age increases.

Based on the background above, a study is needed on the calculation of cost components in the pure premium of multiple life dual-purpose insurance. This research is conducted to maintain the balance between the obligations of the insurance company and the affordability for policyholders. This calculation is also necessary to ensure that the insurance company can pay future benefits, which impacts the high level of solvency and portfolio balance of the company.

2. METHODS

This research aims to calculate the cost components in the gross premium of a multiple life endowment insurance policy by applying a multiple life survival model with an independence assumption, using the Gompertz distribution fitted to the Indonesian Mortality Table IV (TMI IV), combined with actuarial present value calculations and cost structure analysis. The future lifetimes of multiple insured individuals are modeled as continuous random variables whose survival and death probabilities are derived jointly under the assumption that each life is independent. The Gompertz distribution is used to represent each individual's mortality, with its parameters estimated by fitting to the Indonesian Mortality Table IV (TMI IV) to reflect actual population mortality patterns. Joint survival probabilities for multiple lives are then obtained as the product of each individual's survival probability. These are used to calculate the present actuarial values for annuities and term insurance under the multiple life framework. The gross premium is determined by adding various cost components — such as new business costs, premium collection costs, and policy maintenance expenses — to the net premium using standard cost loading factors. To ensure valid and realistic estimates for cost components, a constrained linear regression model is applied, and its performance is evaluated using the coefficient of determination (R^2). The methodological flow from fitting mortality data to determining the gross premium is shown in the flowchart below,

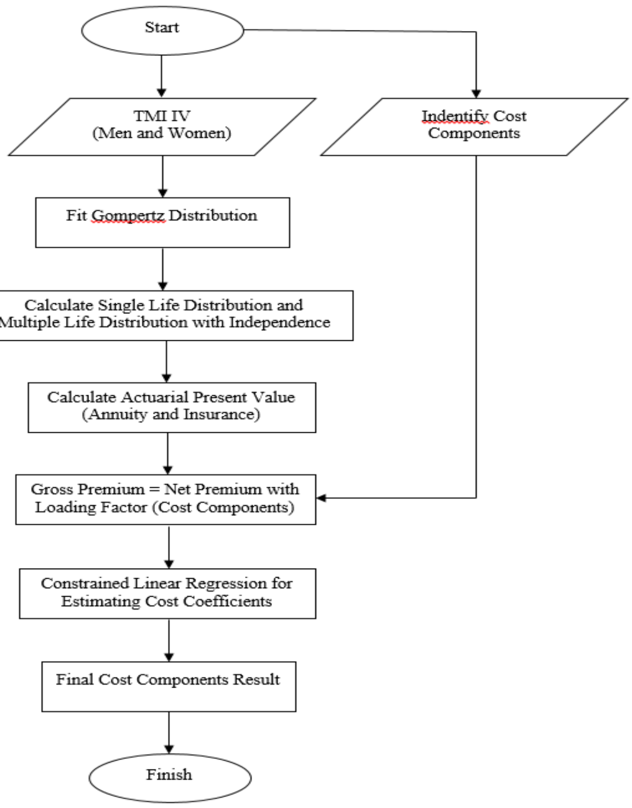

Figure 1. Flowchart of methodology

2.1 Single Life and Multiple Life Model

Given that $T(x)$ is a continuous random variable representing the future lifetime of an individual aged x with x being in the interval $[0, \infty)$. The death of an individual aged x can occur at any time, so the age distribution of x dan $T(x)$ are random variables whose death ages are greater than x . $T(x)$ has a distribution function, a probability function, and a survival function. $F_X(t)$ states the probability that an individual aged x will die before time t, $f_X(t)$ is the probability function of $T(x)$, dan $S_X(t)$ states the probability that an individual aged x will survive at least until time t [5]. The three functions are related as follows:

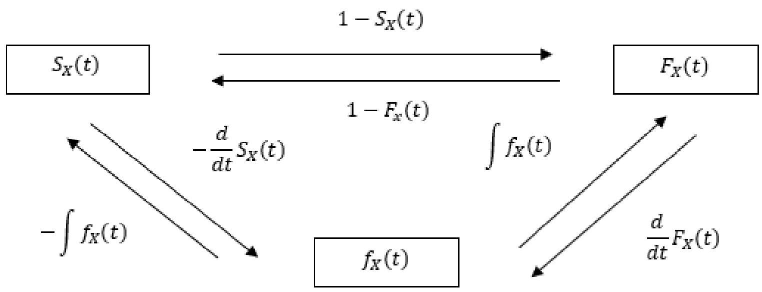

Figure 2. Schematic relationship between $f_X(t)$, $F(x)$, and $S_X(t)$

Then, the survival function and distribution function can also be symbolized as,

$$F_X(t) = \mathrm{P}(T(x) \leq t) = {}_tq_x \tag{1}$$

and

$$S_X(t) = \mathrm{P}(T(x) > t) = 1 - \mathrm{P}(T(x) \leq t) = {}_tp_x \tag{2}$$

Multiple life dual-purpose life insurance is insurance that covers more than one insured person. The probability that both individuals (x) and (y) will survive for t years into the future during the term of the multiple life dual-purpose life insurance policy is called the multiple life survival probability and is denoted as,

$$S_{T(x),T(y)}(t_1, t_2) = P(\{T(x) > t_1\} \cap \{T(y) > t_2\}) = {}_tp_{xy} \tag{3}$$

Assuming independence between x and y, it can be written as follows [10]:

$$ {}_tp_{xy} = {}_tp_x \; {}_tp_y \tag{4}$$

2.2 Gompertz Distribution

The Gompertz distribution is a distribution that can be used to estimate or calculate the probability of someone surviving and dying, as well as the present value of a lifetime annuity. The probability density function (PDF) of Gompertz distribution is

$$f(x) = B\, c^x e^{\left\{\frac{-B}{\ln c}(c^x - 1)\right\}}, 0 \leq x < \omega \tag{5}$$

with $\mathrm{B} > 0, \mathrm{c} > 1, \mathrm{x} > 0,$ through the probability density function, it can be established that the cumulative distribution function (CDF) of Gompertz distribution is as follows,

$$F(x) = 1 - exp\left[-\frac{B}{\ln c}(c^x - 1)\right] \tag{6}$$

The Gompertz distribution has a survival function that can be written as follows,

$$S(x) = exp\left[-\frac{B}{\ln c}(c^x - 1)\right] \tag{7}$$

So that a person's mortality rate function $\mu(x)$, ${}_tp_x$, ${}_tq_x$ can be formed as follows [11],

$$\mu_x = Bc^x \tag{8}$$

$$ {}_tp_x = exp\left[-\frac{Bc^x}{\ln c}(c^t - 1)\right] \tag{9}$$

$$ {}_tq_x = 1 - exp\left[-\frac{Bc^x}{\ln c}(c^t - 1)\right] \tag{10}$$

with, B is the probability of death or failure orccuring; c is the specific growth rate of failure or death

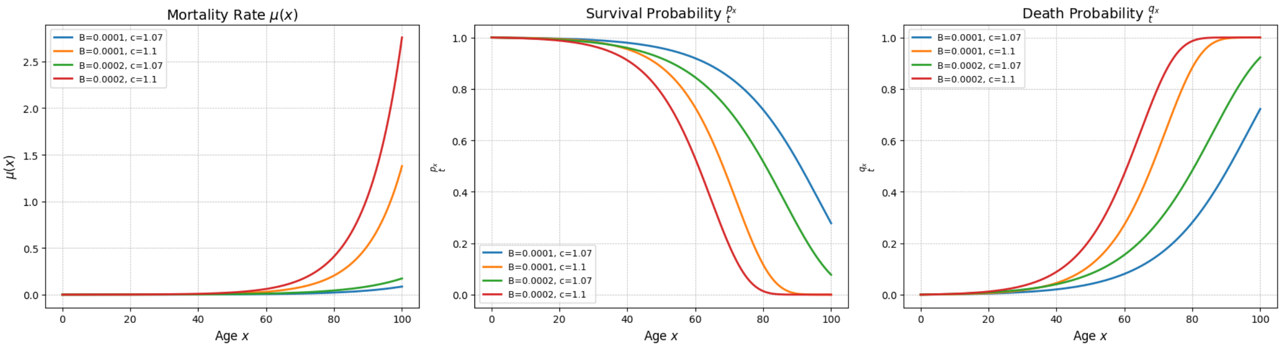

Figure 3. The plots of $\mu(x)$, ${}_tp_x$, ${}_tq_x$ of gompertz distribution

2.3 *Annuity and Insurance Benefit*

The present value is the amount of the initial investment that grows to $(1+i)$ at the end of the first period. The present value can also be referred to as the discount factor denoted by v [10].

$$v = \frac{1}{(1+i)} \tag{11}$$

The present actuarial value of the annuity for dual-purpose multiple life insurance with an annual payment of 1 unit at the beginning of the period can be expressed as,

$$\ddot{a}_{xy:\overline{n}|} = \sum_{k=0}^{n-1} v^k \left({}_tp_{xy} \right) \tag{12}$$

Whereas, the present actuarial value of the annuity for a dual-purpose multiple life insurance with an annual payment of 1 unit at the end of the period can be expressed as [12],

$$a_{xy:\overline{n}|} = \sum_{k=1}^{n} v^k \left({}_tp_{xy} \right) \tag{13}$$

The equation for the actuarial present value in multiple life dual-purpose life insurance can be seen as the same as in term life insurance for n years. The actuarial present value for n-year dual-purpose life insurance is as follows [1]:

$$A_{xy:\overline{n}|} = \sum_{k=0}^{n} v^k \left({}_tp_{xy} \right) \left(\, q_{(x+1)(y+1)} \right) \tag{14}$$

2.4 *Premium Calculation*

The amount of premium received from policyholders is called the gross premium. The gross premium is greater than the net premium, and the difference between the gross premium and the net premium is called the Loading Factor (Cost). The loading factor is usually assumed to be an additional percentage of the net premium. In practice, the context of the loading factor received by life insurance companies is used for the administrative maintenance costs of policyholders, and it also serves as a source of interest income used for reserve purposes.

The formulas for pure premium and gross premium for payment at the beginning of the year are denoted as follows:

$$P_{\overline{xy:n|}} = \frac{A_{\overline{xy:n|}}}{\ddot{a}_{\overline{xy:n|}}} \tag{15}$$

and

$$\vec{P}_{\overline{xy:n|}} = \frac{P_{\overline{xy:n|}}}{1-e} \tag{16}$$

e is percentage of loading factor [1].

Cost components in premium payments

There are various costs that may arise in the calculation of gross premium, including:

1. New closing costs (α_1, α_2).

   New closing costs consist of: insurance supervisor commission fees, field service fees, policy issuance fees, advertising/reclame costs, and sales promotion.

For α_1 : Costs incurred at the beginning of the year, for α_2: Costs incurred at the end of the year

2. Premium collection costs (β).
   Premium collection costs exist throughout the premium coverage period, with the amount of coverage denoted as β
3. Maintenance costs consist of: Electricity, water, building, and so on, applicable at the beginning of each policy year during the coverage period (γ) [13].

From the above cost components, a gross premium model can be formed as follows:

$$\vec{P}_{\overline{xy:n|}} = \frac{(S\times A_{\overline{xy:n|}}) + a_1 P_{\overline{xy:n|}} + a_2 P_{\overline{xy:n|}} v + \beta a_{\overline{xy:n|}} + \gamma \ddot{a}_{\overline{xy:n|}}}{\ddot{a}_{\overline{xy:n|}}} \quad (17)$$

2.5 Linear Regression

Linear regression is one of the statistical approaches used to model the relationship between one dependent variable Y and one or more independent variables $X_1, X_2, \dots, X_k$. This model assumes that the relationship between the variables is linear, which is expressed in the form of a general equation:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k + \varepsilon \quad (18)$$

with:
Y is dependent variable (respons),
$X_1, X_2, \dots, X_k$ is independent variable (predictor),
β_0 is intercept (Y-axis intersection),
β_1 is the regression coefficient that represents the influence of X_i,
ε is error that follows a normal distribution [14].

In practice, not all coefficients in a regression model can take arbitrary values. For example, in certain applications, regression coefficients need to be constrained, such as only taking positive values or being within a specific range. For that, a constrained optimization approach is used. One commonly used technique is bounded optimization, which allows for the search for the best parameters within certain limits.
Optimization problems with constraints can be formulated as:

$$\min f(\theta) \text{ with conditions } \theta_i^{lower} \le \theta_i \le \theta_i^{upper} \quad (19)$$

with
θ is a vector of regression parameters (for example $\beta_1, \beta_2, \dots, \beta_k$)
$f(\theta)$is the objective function that needs to be minimized, such as squared error or other functions [15].

The evaluation of linear regression results can be done using the coefficient of determination, commonly known by the symbol R^2.The Value of R^2 measures how well the regression model explains the variation in the response data Y. Mathematically, R^2 is defined as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y}_i)^2} \quad (20)$$

with
Y_i is the actual value from the data
$\hat{Y}_i$ is the predicted value from the model
$\bar{Y}_i$ is the average of all values Y [16].

3. RESULT AND DISCUSSION

3.1 Transforming the Indonesian Mortality Table IV with the Gompertz Distribution

The Gompertz distribution used in this study is derived from the adjustment of the Indonesian Mortality Table IV. Before estimating the parameters of the Gompertz distribution, the mortality probability (q_x) in the

TMI is first transformed into the form $Y_i = \ln\left(\ln\left(\frac{1}{(1-q_x)}\right)\right)$. The results of the transformation are presented in the following table,

Table 1. Transformation of TMI IV mortality rates

Age	Male	Female	Y_Male	Y_Female
0	0,00524	0,00266	-5,248808	-5,928098
1	0,00053	0,00041	-7,542368	-7,799148
2	0,00042	0,00031	-7,775046	-8,078783
3	0,00034	0,00024	-7,986395	-8,334752
4	0,00029	0,00021	-8,145485	-8,468298
5	0,00026	0,0002	-8,254699	-8,517093
:				
107	0,49429	0,46604	-0,383031	-0,466116
108	0,52467	0,50427	-0,296056	-0,354215
109	0,55733	0,54477	-0.204652	-0,239587
110	0,59244	0,58702	-0,108067	-0,122895

The results of the transformation will then be analyzed using regression analysis in Microsoft Excel. The regression model is formed between the dependent variable (mortality rate) and the independent variable (age). The results of the regression analysis for each gender are obtained as follows:

1. The Regression Analysis of the Indonesian Mortality Table IV for males obtained an R^2 value of 0,956132292 or 95,61% indicating a large variation in a dependent variable (mortality rate). The R Square value indicates that the regression model is very good at predicting the results of the observational data. Regression Equation:

   $Y_L = -9{,}200495355 + 0{,}079163477X$

2. The Regression Analysis of the Indonesian Mortality Table IV for females obtained an R^2 value of 0,957690174 or 95,76% indicating a large variation in a dependent variable (mortality rate). The R Square value indicates that the regression model is very good at predicting the results of the observational data. Regression Equation:

   $Y_L = -9{,}509513076 + 0{,}079063545X$

3.2 Estimation of Gompertz Distribution Parameters

Estimation of the Gompertz distribution parameters can be performed using the equation

$$\ln\left(\ln\left(\frac{1}{(1-q_x)}\right)\right) = x\ln c + \ln\left(\frac{B}{\ln c}\right)(c-1)$$

By using the above equation on the linear regression equation, it is obtained,

a. Estimation of Gompertz distribution parameters for males

$$Y_L = -9{,}200495355 + 0{,}079163477X$$

$$\ln\left(\ln\left(\frac{1}{(1-q_x)}\right)\right) = x\ln c + \ln\left(\frac{B}{\ln c}\right)(c-1)$$

then,

$$\beta_1 = \ln c \leftrightarrow 0{,}079163477 = \ln c \leftrightarrow c = 1{,}082375$$

$$\beta_0 = \ln\left(\frac{B}{\ln c}\right)(c-1) \leftrightarrow -9{,}200495355 = \ln\left(\frac{B}{0{,}07916347}(1{,}082375-1)\right) \leftrightarrow B = 0{,}0000970521$$

b. Estimation of Gompertz distribution parameters for females

$$Y_L = -9{,}509513076 + 0{,}079063545X$$

$$\ln\left(\ln\left(\frac{1}{(1-q_x)}\right)\right) = x\ln c + \ln\left(\frac{B}{\ln c}\right)(c-1)$$

then,

$$\beta_1 = \ln c \leftrightarrow 0{,}079063545 = \ln c \leftrightarrow c = 1{,}082264$$

$$\beta_0 = \ln\left(\frac{B}{\ln c}\right)(c-1) \leftrightarrow -9{,}509513076 = \ln\left(\frac{B}{0{,}079063545}(1{,}082264-1)\right) \leftrightarrow B = 0{,}0000712586$$

Based on the parameters that have been obtained, then can be used to find the probability of death using the equation,

- The male's probability of death

$$q_x = 1 - \exp\left[-\frac{(0{,}0000970521)(1{,}082375)^x}{\ln(1{,}082375)}(1{,}082375-1)\right]$$

- The female's probability of death

$$q_x = 1 - \exp\left[-\frac{(0{,}0000712586)(1{,}082264)^x}{\ln(1{,}082264)}(1{,}082264-1)\right]$$

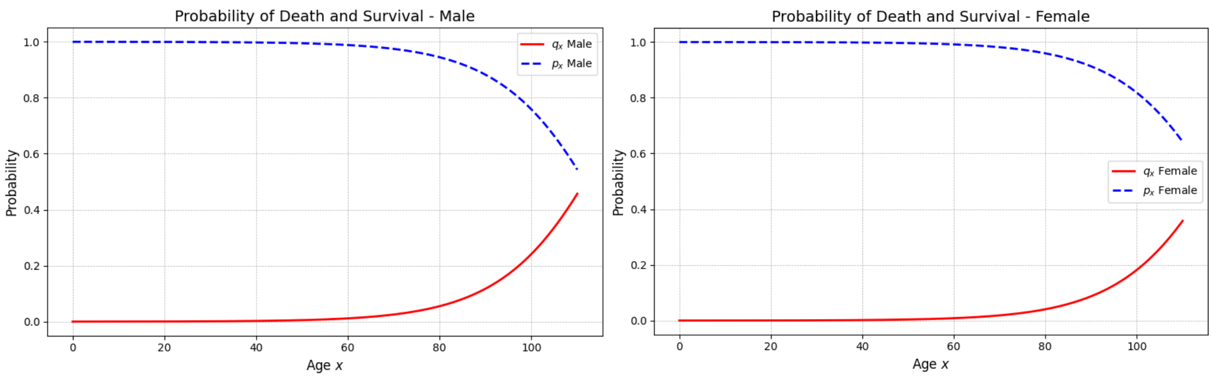

Figure 4. The probability of death and survival for men and women

The plots show that the probability of death (q_x) for both males and females increases exponentially with age, reflecting the typical aging pattern captured by the Gompertz distribution fitted to the given parameters. At younger ages, the probability of death is close to zero and the probability of survival (p_x) remains near one, indicating high survival likelihood in early life. As age approaches the higher range (around age 80 and above), the probability of death rises steeply while the survival probability declines rapidly toward zero, demonstrating the increasing mortality risk in old age. Comparing the male and female curves, it is visible that, with these parameters, females tend to have slightly lower death probabilities and higher survival probabilities at each age, which aligns with general demographic trends showing females having higher life expectancy than males.

Thus, the following table was obtained:

Table 2. Adjustment of TMI IV mortality probability with gompertz distribution

Age	Male (TMI IV)	Female (TMI IV)	Male (Gompertz)	Female (Gompertz)
0	0,00524	0,00266	0,000101	0,000074
1	0,00053	0,00041	0,00011	0,000081
2	0,00042	0,00031	0,000119	0,000087
:				
109	0,55733	0,54477	0,43277	0,337381
110	0,59244	0,58702	0,458654	0,359439
111	1	1	1	1

3.3 Simulation of Cost Component Calculation

This research was conducted with the assumption that the profile of the policyholders consists of male and female couples, with males (x) aged 25-35 years and females (y) aged 21-31 years. The life insurance program followed is a dual-purpose life insurance with a term of 10 years and a sum insured (S) of Rp 5.000.000. The interest rate follows the BI-rate of 5.5%. Additionally, a loading factor (e) of 25% is assumed. The probabilities of death and survival are properties for calculating the present value of annuities paid at the beginning of the year, annuities paid at the end of the year, and the present value of multiple life dual-purpose life insurance.

As an example of a complete calculation with a male $(x = 25)$ and female $(y = 21)$:

1. The present actuarial value of an annuity for a dual-purpose life insurance multiple life policy with an annual payment of 1 unit at the beginning of the period can be expressed as,

$$\ddot{a}_{\overline{25,21:10|}} = \sum_{k=0}^{9} \left(\frac{1}{1,055}\right)^{k} \left({}_{k}p_{25}\ {}_{k}p_{21}\right) = 8,061775900$$

2. The present actuarial value of an annuity for a dual-purpose life insurance multiple life with an annual payment of 1 unit at the end of the period can be expressed as,

$$a_{\overline{25,21:10|}} = \sum_{k=1}^{10} \left(\frac{1}{1,055}\right)^{k} \left({}_{k}p_{25}\ {}_{k}p_{21}\right) = 7,665692612$$

3. The actuarial present value for a two-benefit life insurance policy for years is as follows:

$$A_{\overline{25,21:10|}} = \sum_{k=0}^{10} \left(\frac{1}{1,055}\right)^{k} \left({}_{k}p_{25}\ {}_{k}p_{21}\right)\left(q_{(26)(22)}\right) = 0.586052000$$

4. Next, the net premium and gross premium were obtained as follows:

$$P_{\overline{25,21:10|}} = 5.000.000 \times \frac{A_{\overline{25,21:10|}}}{\ddot{a}_{\overline{25,21:10|}}} = 363.476,00$$

and

$$\vec{P}_{\overline{25,21:10|}} = \frac{P_{\overline{25,21:10|}}}{1-25\%} = 484.634,66$$

The calculations above were conducted for all couples with males aged 26-35 years and females aged 22-31 years, as shown in the table below:

Table 3. Gross premium value, net premium, annuity, and present value of insurance benefit

x	y	$P_{\overline{xy:n\|}}$	$\vec{P}_{\overline{xy:n\|}}$	$A_{\overline{xy:n\|}}$	$a_{\overline{xy:n\|}}$	$\ddot{a}_{\overline{xy:n\|}}$
25	21	363.476,00	484.634,66	0,586052000	7,665692612	8,061775900
26	22	365.043,61	486.724,81	0,588304522	7,661104274	8,058003310
27	23	365.374,54	487.166,06	0,588539714	7,656142581	8,053923332
28	24	365.732,78	487.643,71	0,588794025	7,650777548	8,049511157
29	25	366.120,59	488.160,78	0,589068989	7,644976848	8,044740071
30	26	366.540,39	488.720,52	0,589366257	7,638705640	8,039581266
31	27	366.994,86	489.326,48	0,589687608	7,631926391	8,034003688
32	28	367.486,84	489.982,46	0,590034958	7,624598681	8,027973873
33	29	368.019,46	490.692,62	0,590410370	7,616678997	8,021455775
34	30	368.596,08	491.461,44	0,590816065	7,608120525	8,014410580
35	31	369.220,34	492.293,79	0,591254428	7,598872918	8,006796512

After calculating the actuarial components such as the present value of insurance benefits, the present value of annual premiums, the discount factor, and the life annuity, a mathematical model can be constructed to illustrate the gross premium structure of life insurance products. This model considers the contributions from various cost components, both the benefits paid and the additional costs charged into the premium, resulting in the following gross premium formula:

$$\vec{P}_{\overline{xy:n|}} = \frac{(S \times A_{\overline{xy:n|}}) + \mathrm{a}_1 P_{\overline{xy:n|}} + \mathrm{a}_2 P_{\overline{xy:n|}} v + \beta a_{\overline{xy:n|}} + \gamma \ddot{a}_{\overline{xy:n|}}}{\ddot{a}_{\overline{xy:n|}}}$$

This model is then estimated using a linear regression approach with bounded optimization through the Python programming language. The optimization process is conducted with the constraint that all parameters must be positive and cannot be zero, to maintain the consistency of actuarial practice logic. The simulation results show that the obtained coefficient values are $a_1 = 0{,}97074, a_2 = 0{,}912217, \beta = 19.344{,}46, \gamma = 20.105{,}23$. The interpretation of these results shows that the component of new closure costs (a_1, a_2) relative to the pure premium approaches full value (around 90%–97%). For example, for a couple where the man is 25 years old and the woman is 21 years old, the acquisition cost paid at the beginning of year a_1 is $0{,}97074 \times P_{\overline{25,21:10|}} = 0{,}97074 \times \text{Rp } 363.476{,}00 = \text{Rp } 352.840{,}692$ and the acquisition costs are paid at the end of the year a_2 is $0{,}912217 \times P_{\overline{25,21:10|}} \times \frac{1}{1{,}055} = 0{,}912217 \times \text{Rp } 363.476{,}00 \times \frac{1}{1{,}055} =$ Rp 314.283,399. Whereas, the premium collection cost (β) is Rp 19.344,46 and the maintenance cost, which includes electricity, water, building, and so on, applicable at the beginning of each policy year during the coverage period (γ) is Rp 20.105,23.

Then, the simulation conducted resulted in a coefficient of determination R^2 of 98,46%, indicating that this model has a very high goodness of fit. This means that more than 98% of the variation in the gross premium data can be explained by the model. This indicates that this formulation approach is not only mathematically valid but also highly effective in representing the actual cost components in the calculation of the gross premium for term life insurance products.

4. CONCLUSIONS

Based on the research results, it can be concluded that the probability of death for males and females was successfully estimated by fitting the Gompertz distribution to the Indonesian Mortality Table IV, producing mortality parameters that align with actual population patterns. The gross premium model formed in this study includes detailed actuarial cost components: an initial acquisition cost ($\alpha_1 = 0{,}9707$); an end-of-first-year acquisition cost ($\alpha_2 = 0{,}912217$); a premium collection cost ($\beta = \text{Rp } 19.344{,}46$); and an annual maintenance cost ($\gamma = \text{Rp Rp } 20.105{,}23$). All parameters were estimated using a linear regression approach with bounded optimization and yielded positive, logically consistent results in accordance with actuarial practice. The simulation using Python produced a coefficient of determination of $R^2 = 98{,}46\%$, indicating that the model has excellent predictive power and can represent the cost structure accurately in calculating premiums for multiple life endowment insurance products. This study highlights the advantage of combining mortality modeling with explicit cost component estimation. However, it is limited by its use of a single mortality table and the assumption of independence between insured lives. Future research is recommended to test alternative mortality models, incorporate possible dependencies among lives, and validate the approach with larger and more diverse datasets to enhance the model's applicability and accuracy.

5. ACKNOWLEDGMENTS

The authors would like to express their sincere gratitude to Sekolah Tinggi Manajemen Asuransi (STMA) Trisakti for the funding and support provided for the completion of this research.

6. REFERENCES

[1] D. C. M. Dickson, M. R. Hardy, and H. R. Waters, *Actuarial Mathematics for Life Contingent Risks*. Cambridge University Press, 2009. doi: 10.1017/CBO9780511800146.

[2] E. W. Frees, J. Carriere, and E. Valdez, "Annuity Valuation with Dependent Mortality," *J. Risk Insur.*, vol. 63, no. 2, pp. 229–255, 1996, [Online]. Available: http://www.jstor.org/stable/253744

[3] K. N. Sari, A. Febrisutisyanto, R. Deautama, N. Azirah, and P. Mahani, "Exploring Multivariate Copula Models and Fuzzy Interest Rates in Assessing Family Annuity Products," *JTAM (Jurnal Teor. dan Apl. Mat.*, vol. 8, no. 2, p. 555, 2024, doi: 10.31764/jtam.v8i2.17467.

[4] D. N. Trisnawati, I. N. Widana, and K. Jayanegara, "Analisis Komponen Biaya Asuransi Jiwa Dwiguna (Endowment)," *J. Mat.*, vol. 04, no. 1, pp. 1693–1394, 2014.

[5] N. L. Bowers, H. U. Gerber, J. C. Hickman, D. A. Jones, and C. J. Nesbitt, *Actuarial Mathematics*, Second. The Society of Actuaries, 1997.

[6] F. Castellares, S. C. Patricio, and A. J. Lemonte, "On Gamma-Gompertz-Makeham Assurances and Life Annuities," *Austrian J. Stat.* , vol. 53, no. 4, pp. 1–24, 2024, doi: 10.17713/ajs.v53i4.1785.

[7] K. Alfiana and A. I. A. Himayati, "Penentuan Premi Asuransi Jiwa Gabungan Dwiguna dengan Asumsi Mortalita Gompertz dan Pengaruh Nilai Tukar terhadap Tingkat Bunga," *Sq. J. Math. Math. Educ.*, vol. 2, no. 2, p. 133, 2020, doi: 10.21580/square.2020.2.2.6411.

[8] R. Ruhiyat, W. Erliana, K. Lamberto, and E. Ardelia, "Last-Survivor Insurance Premium and Benefit Reserve Calculation using Gamma-Gompertz Mortality Law," *J. Mat. Integr.*, vol. 18, no. 1, p. 9, 2022, doi: 10.24198/jmi.v18.n1.38678.9-18.

[9] W. Syaftira, R. Lestari, and Y. Yanita, "Penentuan Premi Asuransi Jiwa Dwiguna Dengan Hukum De Moivre Dan Hukum Gompertz," *J. Mat. UNAND*, vol. 6, no. 3, p. 112, 2017, doi: 10.25077/jmu.6.3.112-117.2017.

[10] T. Futami, *Matematika Asuransi Jiwa*. Incorporated Foundation, Oriental Life Insurance Cultural Development Center, 1993.

[11] F. N. Hidayat, R. Cahyandari, and A. S. Awalluddin, "Penerapan Hukum Mortalita Gompertz untuk Perhitungan Dana Tabarru' dengan Metode Cost of Insurance," *Kubik J. Publ. Ilm. Mat.*, vol. 4, no. 1, pp. 156–162, 2019, doi: 10.15575/kubik.v4i1.5676.

[12] K. N. Sari, R. Deautama, and A. Febrisutisyanto, "the Benefits of Family Annuity Calculation With Vine'S Copula and Fuzzy Interest Rate," *BAREKENG J. Ilmu Mat. dan Terap.*, vol. 17, no. 4, pp. 2461–2470, 2023, doi: 10.30598/barekengvol17iss4pp2461-2470.

[13] S. Tamalonggehe, A. Rindengan, and T. Manurung, "Perhitungan Premi Netto Tahunan Dalam Menganalisis Komponen Biaya Pada Perusahaan Asuransi Jiwa Bumiputera," *d'CARTESIAN*, vol. 6, no. 2, p. 63, 2017, doi: 10.35799/dc.6.2.2017.17836.

[14] M. H. Kutner, C. J. Nachtsheim, J. Neter, and W. Li, *Applied Linear Statistical Models*, Fifth., vol. 138, no. 2. McGraw-Hill/Irwin, 2005. doi: 10.2307/2984653.

[15] D. C. Montgomery, E. A. Peck, and G. G. Vining, *Introduction to Linear Regression Analysis*, Fifth. New Jersey: John Wiley & Sons, Inc., 2012.

[16] D. Chicco, M. J. Warrens, and G. Jurman, "The coefficient of determination R-squared is more informative than SMAPE, MAE, MAPE, MSE and RMSE in regression analysis evaluation," *PeerJ Comput. Sci.*, vol. 7, pp. 1–24, 2021, doi: 10.7717/PEERJ-CS.623.