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Catastrophe Reinsurance Single Premium Valuation Model Based on Indonesia's Earthquake Data

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Abstract
Indonesia's position along the Pacific Ring of Fire makes it highly vulnerable to catastrophic earthquakes, creating significant financial exposure for insurers through simultaneous surges in life and health insurance claims. This study develops a comprehensive valuation model for catastrophe reinsurance contracts using advanced statistical techniques to assess extreme risks and their interdependencies. The model integrates three key approaches: (1) the Peaks Over Threshold (POT) method with Generalized Pareto Distribution to analyze extreme losses from deaths and injuries, (2) copula theory (specifically Gumbel copula, demonstrating superior fit for upper-tail dependence) to capture dependency structures between deaths and injuries, and (3) Monte Carlo simulations to project future event frequencies and financial impacts. Utilizing Indonesian seismic data from 1979 to 2025, while excluding extreme outlier events, reinsurance premiums are estimated as the expected present value of potential claims, employing the Fundamental Theorem of Asset Pricing. Applying realistic assumptions—including a Rp15 billion retention limit for the primary insurer, average claims of Rp500 million per life and Rp15 million per injury, and coverage for 5% and 7% of the population for life and health policies respectively, alongside a 5.75% discount rate (BI rate 2025) through 10,000 Monte Carlo simulations—a single reinsurance premium of Rp17,395,932,554 is calculated. These results demonstrate how advanced statistical methods can effectively quantify catastrophe risk transfer, providing insurers with an actuarially sound pricing framework for managing low-frequency, high-severity earthquake exposures. However, a limitation of this study includes the exclusion of the 2004 mega-disaster, which may lead to an underestimation of worst-case scenarios, and the use of fixed assumptions for insurance coverage and claim values, which may not fully reflect real-world variability. Despite these limitations, this approach offers a valuable framework for managing earthquake-related risks in Indonesia's reinsurance market.

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1. INTRODUCTION

Indonesia, positioned directly along the volatile Pacific Ring of Fire, is one of the most seismically active regions globally. This vulnerability stems from its unique geographical location at the intersection of several major tectonic plates, including the Asian, Australian, and Pacific Plates. The constant movement of these plates results in an alarmingly high frequency of earthquakes, which consistently inflict substantial loss of life, widespread property damage, and significant economic disruption across the archipelago. These seismic events pose profound and multifaceted risks to both businesses and individuals, extending beyond immediate physical destruction to contribute to long-term financial challenges [1].

In the aftermath of catastrophic earthquakes, a profound and immediate financial strain is placed upon the insurance sector. A significant surge in insurance claims inevitably occurs, encompassing various types of coverage. Crucially, this includes a substantial increase in life insurance payouts for earthquake-related deaths, alongside health insurance claims for the injured, and property damage claims. For insurers offering diverse coverage types, managing the sheer volume and cumulative cost of these claims can severely strain financial resources, often leading to substantial losses. This financial pressure can escalate to the point of bankruptcy, particularly when traditional catastrophe models, often built on limited historical data, fail to accurately estimate the true extent of the risks [2]. Consequently, insurers may find themselves critically overexposed, unable to manage the aggregated cost of claims during large-scale disasters, thus jeopardizing their solvency and their ability to fulfill policyholder obligations.

To mitigate the severe financial impact of such large-scale disasters, insurance companies widely employ reinsurance. This critical mechanism involves transferring a portion of the underwriting risk to another insurer, thereby significantly reducing the primary insurer's exposure to catastrophic claims. Reinsurance is not merely a financial transaction; it is a fundamental tool that helps maintain market stability by distributing risk more broadly between primary insurers and reinsurers. This collaborative risk-sharing model ensures that primary insurers can continue fulfilling their policyholder claims even in the face of devastating, large-scale catastrophic events, thereby safeguarding the integrity of the insurance market.

However, earthquake events present unique challenges for risk modeling. They are characterized by extreme variability in their magnitude, intensity, and location, and involve multiple interdependent risks that can cascade across different insurance lines. This inherent complexity makes them notoriously difficult to model effectively using traditional actuarial methods. Conventional methods of reinsurance valuation often prove inadequate when confronted with the multifaceted nature of catastrophic events, especially when considering the intricate interdependencies between different types of claims such as life insurance, property, and health.

The research gap in the existing literature is a significant one: there is an identified need for the development and application of advanced statistical models specifically tailored for life insurance reinsurance pricing in regions highly susceptible to earthquakes, such as Indonesia. While advanced statistical models, including the Peaks Over Threshold (POT) model and copula techniques, have been recognized as essential tools for assessing extreme events and capturing dependencies [3], and Monte Carlo simulations are highly valued for pricing catastrophe bonds by generating multiple disaster scenarios [4], their integrated application and empirical validation for life insurance products in the context of Indonesian earthquake data, for the purpose of developing more precise and robust reinsurance pricing models, remain largely underexplored. Existing models often fail to fully capture the unique characteristics of life insurance claims in a catastrophic earthquake scenario, including mortality rate spikes, long-term health complications leading to claims, and the potential for a large number of simultaneous claims.

This research, therefore, aims to bridge this critical gap. The ultimate goal is to significantly improve life insurance reinsurance pricing by offering sophisticated models that accurately reflect the intricate and complex nature of catastrophic risks stemming from seismic activity. By applying and integrating advanced statistical techniques such as the POT model, copula techniques to model the dependencies between various claim types affecting life insurance (e.g., direct mortality vs. long-term health complications), and Monte Carlo simulations to generate a multitude of realistic disaster scenarios, this study seeks to enhance the ability of both primary insurers and reinsurers to more accurately estimate potential losses and financial exposures specifically related to life insurance portfolios. By leveraging granular data from past Indonesian earthquakes and a deep understanding of their impact on life insurance claims, this research will contribute to more precise pricing models for catastrophe reinsurance contracts. This will, in turn, lead to enhanced decision-making regarding risk pricing, coverage limits, and capital allocation, ultimately fostering a more resilient financial ecosystem for the life insurance sector in an increasingly volatile environment. In regions like Indonesia, where seismic activity is frequent and the stakes for human life are exceptionally high, such refined models are not merely academic exercises but crucial tools for navigating the complex and vital landscape of life insurance catastrophe risk management.

2. METHODS

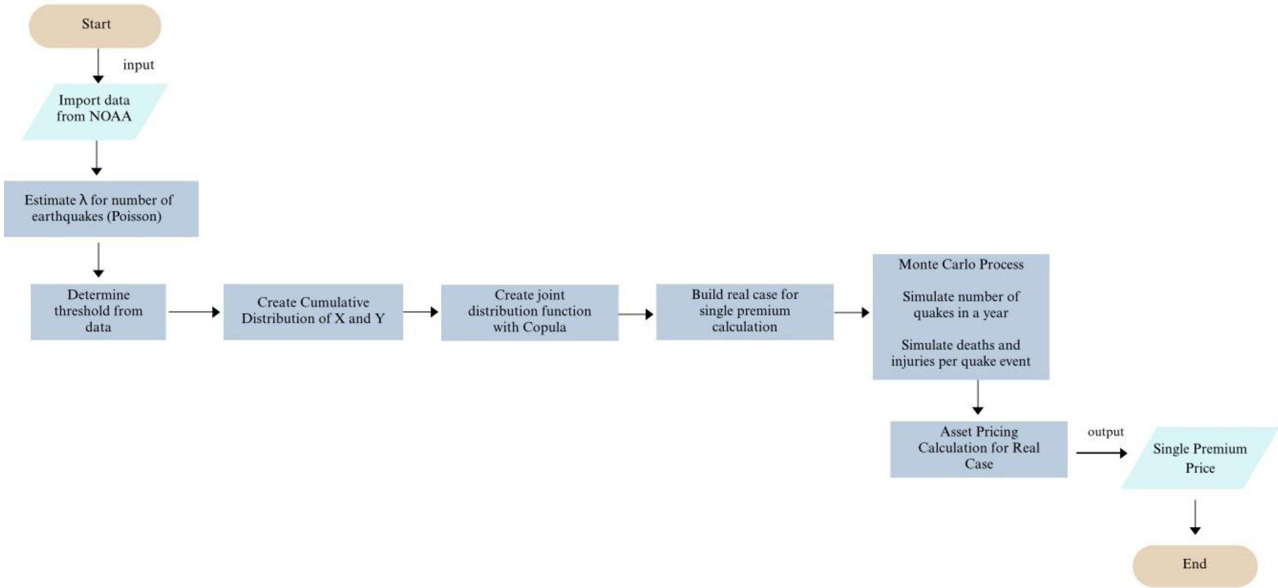

Figure 1. Methods Flowchart

Figure 1 illustrates the comprehensive methodology employed in this research to develop a robust single premium valuation model for catastrophe reinsurance contracts, specifically targeting life and health insurance policies in Indonesia. The process begins with data input from NOAA, capturing historical earthquake events, followed by the estimation of the frequency of earthquakes per year using a Poisson distribution to model potential future occurrences. The core of the model then focuses on the extreme impacts of these events: X represents the total number of deaths, directly impacting life insurance claims, and Y represents the total number of injuries, relevant for health insurance claims. A threshold is determined from the historical data for both X and Y to identify extreme events, and their cumulative distribution functions (CDFs) are created using the Peaks Over Threshold (POT) method with the Generalized Pareto Distribution (GPD). To accurately capture the inherent co-occurrence and interconnectedness of deaths and injuries in catastrophic events, a joint distribution function is created using copula theory (specifically the Gumbel copula) to model the dependency structure between X and Y. This integrated probabilistic framework then feeds into a Monte Carlo simulation process, which first simulates the number of quakes in a year based on the Poisson distribution, and then generates potential joint outcomes of deaths (X) and injuries (Y) for each simulated extreme earthquake event. These simulated outcomes are crucial inputs for the asset pricing calculation for a real-case scenario, which ultimately determines the single reinsurance premium price. This final premium represents the expected present value of potential claims that exceed the primary insurer's specified retention limit, providing an actuarially sound basis for transferring catastrophic life and health insurance risks. A detailed breakdown of each step, from data input to premium calculation through Monte Carlo simulations, is provided below.

2.1 Peaks-Over-Threshold

The Peaks Over Threshold (POT) method is a widely used technique in Extreme Value Theory (EVT) for identifying and modelling extreme observations. POT focuses on all data points that exceed a predefined high threshold. This allows for more efficient use of available data, particularly in contexts such as catastrophe reinsurance where extreme losses are rare but highly impactful.

In the POT framework, a threshold value m is selected, and any observation X such that $X > m$ is considered an extreme event. The exceedances over this threshold are then modeled using the Generalized Pareto Distribution (GPD). For a sufficiently high threshold, the distribution of these exceedances can be well approximated by the GPD [5]. This makes the POT method especially useful in modelling the tail behavior of distributions, such as those found in catastrophe reinsurance losses. The cumulative distribution function (CDF) of the GPD is given by:

$$F_{\xi,\beta}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\beta}\right)^{-\frac{1}{\xi}}, \xi \neq 0; \\ 1 - \exp\left(-\frac{y}{\beta}\right), \xi = 0, \end{cases} \tag{1}$$

and the probability density function (pdf) for GDP follows:

$$f_{\xi,\beta}(y) = \begin{cases} \frac{1}{\beta}\left(1 + \frac{\xi y}{\beta}\right)^{-1-\frac{1}{\xi}}, \xi \neq 0; \\ \frac{1}{\beta}\exp\left(-\frac{y}{\beta}\right), \xi = 0, \end{cases} \tag{2}$$

where $\beta > 0$ and $y \geq 0$ if $\xi \geq 0, 0 \leq y \leq -\beta/\xi$ if $\xi < 0, \xi$ and β are the shape parameter and scale parameter, correspondingly.

One commonly applied method for identifying extreme values in a dataset is the percentage-based threshold selection. This approach defines a fixed proportion of the highest data points as extreme values. A frequently used standard is to consider the top 5% to 10% of the data as extreme.

Furthermore, having identified and characterized extreme values for deaths and injuries using the Peaks Over Threshold method, it becomes imperative to simulate various future scenarios for a comprehensive assessment of the overall financial exposure in reinsurance. To achieve this, Monte Carlo simulation serves as a crucial methodology, facilitating the robust probabilistic modeling of diverse outcomes by incorporating the stochastic factors of earthquake frequency and the severity of extreme events derived from the POT analysis.

2.2 Monte Carlo Simulation

The Monte Carlo Simulation is a way to model the probability of different outcomes in a process that cannot be easily predicted due to the random factors that could affect the outcomes happening in the future [6]. Because of that, the simulation is often referred to as a multiple probability simulation. The Monte Carlo simulation can be applied in various problems from different fields such as investments, business, physics, and economics. There are often uncertainties in making forecasts or estimates from previous data, but this simulation explains the impact of risk and uncertainty in prediction and forecasting models. Monte Carlo enables accurate simulations involving randomness and known factors.

The Monte Carlo analysis consists of input variables, output variables, and a mathematical model. In a programming system, it provides independent variables into the mathematical model, simulates them, and produces dependent variables. The input variables are random factors that affect the outcome of the Monte Carlo simulation. In the case of earthquake occurrences, they factor to be considered are the number of earthquakes in the past years of the dataset. The output variable is the result of the Monte Carlo simulation. The number of earthquakes happening in the past years would give a forecast of number of earthquakes that could happen in the future years.

The mathematical model would describe the relationship between the input and output variables. The average number of earthquakes occurring every year is found from the dataset of past earthquakes. However, even though the average is known, future earthquake occurrence each year cannot always occur average times as they occur randomly. So, the number of earthquakes that is predicted to happen each year in the future would be a random number generated from the mean value and standard deviation [7].The results of the simulation can be shown in a histogram that models the relationship between number of earthquakes per year as the horizontal axis, and the number of years that earthquake amount occurred as the vertical axis. The histogram would form a standard normal distribution. In a normal distribution, the mean number of earthquakes would happen in the greatest number of years. While the number of years where a certain number of earthquakes happen becomes lesser as the number of earthquakes increases or decreases.

The poison distribution calculates the probability that a given number of events will occur in a fixed interval of time when they occur at a known average rate [8]. The probability of a given number of earthquakes happening in a year can be calculated using Poisson distribution. The probability is highest at the mean or median value. And the probability of a higher or lower number of earthquakes than the mean becomes smaller the more it increases or decreases. The probability can be calculated using the formula below:

$$P(X = z) = \frac{e^{-\lambda}\lambda^{z}}{z!}$$

where,

a. λ is the mean number of times an event occurs in a given time interval
b. z is the number of times an event is occurring in a given time interval.

As Monte Carlo simulations generate a multitude of event outcomes for deaths and injuries, accurately capturing their intricate interdependencies is crucial, given these variables are rarely independent in real-world catastrophic events. Therefore, copulas are utilized to integrate and preserve the statistical dependence observed between simulated deaths and injuries, ensuring a more realistic and robust assessment of the combined risk within the simulation framework.

2.3 Copulas

A copula is a multivariate joint distribution function where each variable has a uniform marginal probability distribution on the interval [0,1] [9]. The primary function of a copula is to model the dependencies between two or more random variables, separate from their marginal distributions. An advantage in using copulas is that it does not require the variables to have identical and normally distributed marginal distributions. Copulas are fundamentally based on Sklar's theorem [10], which states that:

Sklar's Theorem. *Let F be an n-dimensional distribution function with marginals $F_1, F_2, \dots, F_n$ Then there exists a copula function C , such that for all $(X_1, X_2, \dots, X_n) \in \mathbb{R}^n$ then*

$$F(X_1, X_2, \dots, X_n) = C\big(F_1(X_1), F_2(X_2), \dots, F_n(X_n)\big) \tag{3}$$

In case of two dependent variables, X and Y, the cumulative distribution function can be written as $C(u, v)$, where $u = F_x(X)$ and $v = F_y(Y)$.

2.3.1 Clayton Copulas

The Clayton copula is a type of copula that captures lower-tail dependence, meaning it exhibits stronger dependence between variables for extreme low values compared to high values. The Clayton copula's cumulative distribution function (CDF) for two random variables is given by:

$$C^{Cl}(u, v) = (u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}}, 0 < \theta < \infty \tag{4}$$

where,

a. u and v are the marginal cumulative distribution functions (CDFs) of the two variables, each uniformly distributed on [0,1],
b. θ is the dependence parameter, controlling the strength and direction of dependence.

The probability density function (PDF) of the Clayton copula for two random variables [11] is given by:

$$C^{Cl}(u, v) = (\theta + 1)(u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}-2}\,(uv)^{-\theta-1} \tag{5}$$

where,

a. u and v are the marginal cumulative distribution functions (CDFs) of the two variables, each uniformly distributed on [0,1],
b. θ is the dependence parameter, where higher values indicate stronger lower-tail dependence.

The conditional copula function for the Clayton copula [12] is given by:

$$C^{Cl}(v|u) = \left[1 + u^{\theta}\left(v^{-\theta} - 1\right)\right]^{-1-\left(\frac{1}{\theta}\right)} \tag{6}$$

And the inverse:

$$C^{Cl^{[-1]}}(v|u) = \left[\left(v^{-\frac{\theta}{1+\theta}} - 1\right)u^{-\theta} + 1\right]^{-\frac{1}{\theta}} \tag{7}$$

For the Clayton copula, Kendall's Tau is given by:

$$\tau^{Cl} = \frac{\theta}{\theta + 2}\,, \theta \,\epsilon\, (0, \infty) \tag{8}$$

where,

a. θ is the dependence parameter of the Clayton copula.
b. τ^{Cl} increases with θ, indicating stronger lower-tail dependence as $\tau^{Cl} \to 1$.

2.3.2 Gumbel Copulas

The Gumbel copula is an Archimedean copula that models upper-tail dependence, meaning it captures stronger correlations between extreme high values of two variables. The cumulative distribution function (CDF) of the Gumbel copula for two random variables is given by:

$$C^{Gu}(u,v) = \exp\left\{-\left((-\,ln\,u)^{\theta} + (-\,ln\,v)^{\theta}\right)^{\frac{1}{\theta}}\right\}, 1 \le \theta < \infty \tag{9}$$

where,

a. u and v are the marginal CDFs of the two variables (each uniformly distributed in [0,1]),
b. θ is the dependence parameter, controlling the strength of upper-tail dependence.

The probability density function (PDF) of the Gumbel copula for two random variables is given by:

$$c^{Gu}(u,v) = C^{Gu}(u,v)\frac{[(-\ln u)(-\ln v)]^{\theta-1}}{uv}\left[(-\ln u)^{\theta} + (-\ln v)^{\theta}\right]^{\frac{2}{\theta}-2}$$
$$\left\{(\theta - 1)\left[(-\ln u)^{\theta} + (-\ln v)^{\theta}\right]^{-\frac{1}{\theta}} + 1\right\} \tag{10}$$

Conditional copula for Gumbel copulas with two random variables is listed below:

$$C^{Gu}(v|u) = \frac{1}{u}\exp\left\{-\left((-\,ln\,u)^{\theta} + (-\,ln\,v)^{\theta}\right)^{\frac{1}{\theta}}\right\}\left[1 + \left(\frac{\ln u}{\ln v}\right)^{\theta}\right]^{-1+\frac{1}{\theta}} \tag{11}$$

Gumbel copulas lack a closed-form solution for the inverse of their conditional copula function.

For the Gumbel copula, Kendall's Tau is given by:

$$\tau^{Gu} = 1 - \frac{1}{\theta}\,, \theta \,\epsilon\, (0, \infty) \tag{12}$$

where,

a. θ is the dependence parameter of the Gumbel copula.
b. τ^{Gu} increases with θ, indicating stronger upper-tail dependence as $\tau^{Gu} \rightarrow 1$.

3. RESULT AND DISCUSSION

The dataset for this valuation model comprises Indonesian earthquake data obtained from the National Centers for Environmental Information (NOAA) covering the period from 1979 to 2025 (as of April 29). In constructing the model, we focus specifically on events where the total number of people affected ranges from 1 to 50. This range is selected to capture typical catastrophic events while excluding extreme outliers. Notably, we exclude the 2004 Aceh earthquake from the dataset due to its exceptionally high impact, which would disproportionately skew the model. The valuation model will be specifically designed to leverage the "total deaths" (X) and "total injuries" (Y) variables to estimate the expected losses for life and health insurances products, which are then expected to be covered by the catastrophe reinsurance contract.

3.1 Descriptive Statistics

Table 1 shows that the average number of deaths and injuries resulting from catastrophic events is approximately 142 and 524, respectively. Despite relatively low median values of 4 for deaths and 28 for injuries, the data contains several large observations, with maximum values reaching 4,340 for deaths and 10,679 for injuries. This wide range, along with high skewness values of 6.19 for deaths and 5.01 for injuries, supported by the kurtosis values, confirms the heavy-tailed nature of the data, suggesting the presence of rare but significant events. These statistical properties highlight the need for extreme value theory in modelling such variables, particularly in disaster reinsurance, where accurate risk evaluation is crucial.

Table 1. Descriptive statistic of total deaths and total injuries

Variable	Mean	Standard Deviation	Min	Q1	Q2	Q3	Max	Kurtosis	Skewness
Total Deaths (X)	141.8276	607.8151	1	1	4	23	4340	41.6074	6.1904
Total Injuries (Y)	524.3103	1729.5682	1	8.5	28	237.5	10679	25.9211	5.0074

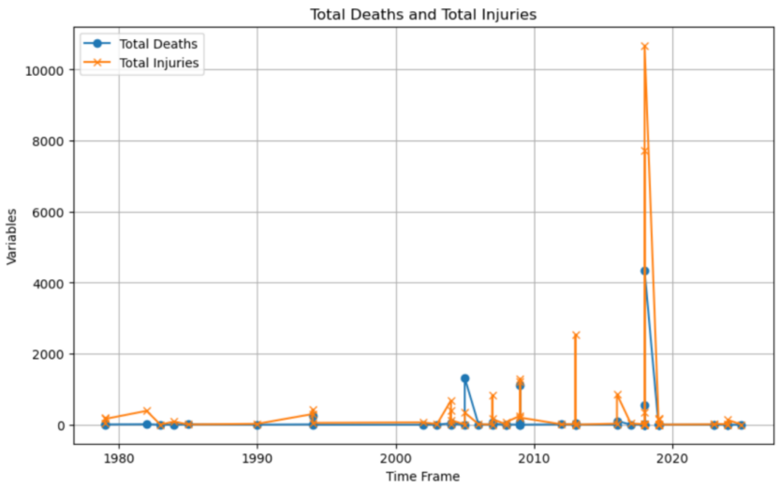

Figure 2. Total deaths and injuries data

Figure 2 illustrates the total deaths and total injuries resulting from earthquake events in Indonesia from 1979 to 2025. For much of the early period (1979 to early 2000s), most earthquake events resulted in relatively low casualties, rarely exceeding a few hundred individuals. However, the graph displays notable spikes in later years, particularly during the 2010s, where some events caused thousands of deaths and injuries, reflecting the occurrence of major catastrophes. These spikes highlight the presence of extreme events that significantly deviate from the general pattern and are essential in risk modelling.

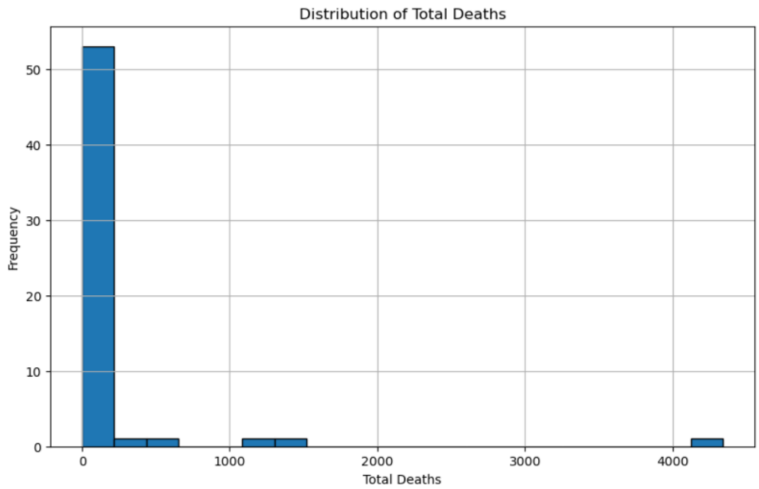

Figure 3. Distribution of total deaths

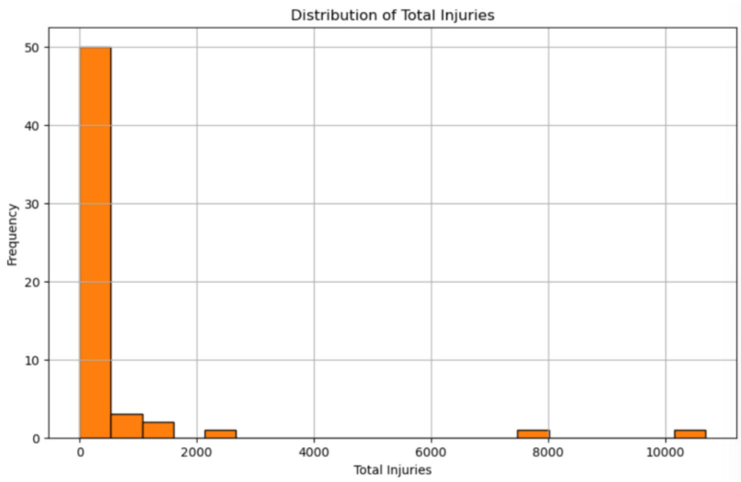

Figure 4. Distribution of total injuries

The histograms illustrate the distribution of total deaths and injuries caused by earthquakes. From Figure 3, it is evident that most earthquakes result in relatively low death counts, with over 70 instances concentrated in the lowest range. However, a small number of events show extremely high death tolls, with one event exceeding 4,000 deaths. This indicates a right-skewed distribution, where the majority of earthquakes are not deadly, but a few rare cases cause catastrophic loss of life.

Similarly, Figure 4 shows a comparable pattern with total injuries. The majority of earthquakes caused fewer than 2,000 injuries, but there are notable outliers, with one causing over 10,000 injuries and another close to 8,000. This also reflects a heavily right-skewed distribution. These patterns suggest that while high-impact earthquakes are infrequent, their effects are disproportionately severe.

3.2 Poisson Parameter Estimation

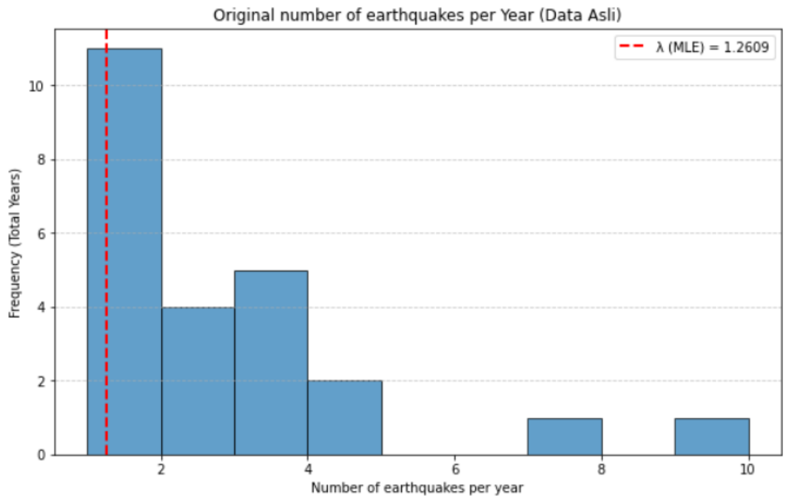

Figure 5. Number of earthquakes per year

Based on the historical earthquake data in Indonesia, the number of earthquakes per year was analysed and assumed to follow a Poisson distribution, which is appropriate for modelling the occurrence of earthquakes over a certain time interval. The parameter of the Poisson distribution, λ (lambda), representing the average number of earthquakes per year was estimated using the Maximum Likelihood Estimation (MLE) method. The estimated λ (MLE) = 1.2609, this indicates that on average, approximately 1.2609 earthquakes occur per year in the dataset.

3.3 Threshold and POT Model Parameter Estimation

Table 2 presents the estimation of the Peak Over Threshold (POT) model parameters used to analyze extreme values of earthquake-related deaths and injuries. The thresholds $\boldsymbol{m_x}$ and $\boldsymbol{m_y}$ represent the minimum values above which the total deaths $(\boldsymbol{X})$ and injuries $(\boldsymbol{Y})$ are considered extreme.

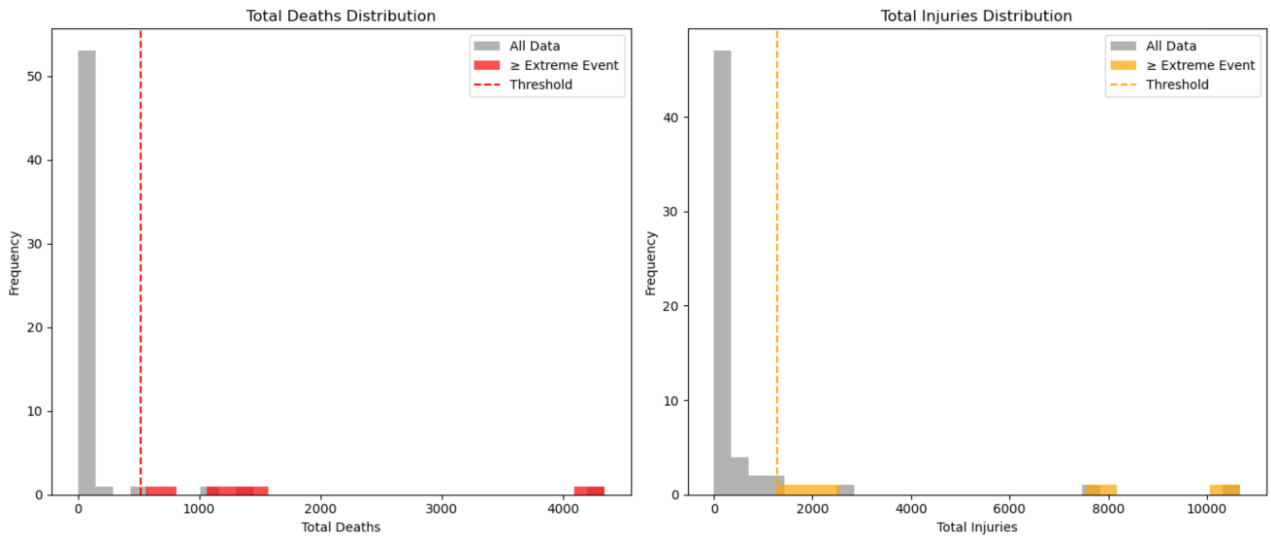

Figure 6. Extreme event distribution for total deaths and injuries

The shape parameters $\boldsymbol{\xi_x}$ and $\boldsymbol{\xi_y}$ determine the heaviness of the tail of the distribution, indicating the likelihood of very large extreme events. The scale parameters $\boldsymbol{\beta_x}$ and $\boldsymbol{\beta_y}$ reflect the spread or dispersion of these extreme values.

Table 2. Threshold and POT parameters result estimation

Parameters	Estimation
m_x	517
ξ_x	0.7073562
β_x	447.822
m_y	1285
ξ_y	0.0071078
β_y	3816.675

From Table 2 and the empirical cumulative distribution function $\left(\hat{F}(x)\right)$ can be obtained:

$$F_x(x) = \begin{cases} (1 - \hat{F}_x(517)\, H_{\hat{\xi}_x,\hat{\beta}_x}(x - 517) + \hat{F}_x(517), x > 517 \\ \hat{F}(x), x \leq 517 \end{cases}$$

with

$$H_{\hat{\xi}_x,\hat{\beta}_x}(x - 517) = 1 - \left(1 + 0.7073562\,\frac{(\mathrm{x} - 517)}{447.822}\right)^{-\frac{1}{0.7073562}}.$$

and

$$F_y(y) = \begin{cases} (1 - \hat{F}_y(1285)\, H_{\hat{\xi}_y,\hat{\beta}_y}(y - 1285) + \hat{F}_y(1285), y > 1285 \\ \hat{F}(y), y \leq 1285 \end{cases}$$

with

$$H_{\hat{\xi}_y,\hat{\beta}_y}(y - 1285) = 1 - \left(1 + 0.0071078\,\frac{(\mathrm{y} - 1285)}{3816.675}\right)^{-\frac{1}{0.0071078}}.$$

where, $F_x(x)$ the cumulative distribution functions for deaths and $F_y(y)$ for injuries.

3.4 Copula Parameter Estimation

The parameter estimation for Clayton copulas and Gumbel copulas are in Table 3.

Table 3. Copulas parameter estimation

Copula	θ
Gumbel	2.057
Clayton	1

By substituting the estimation result for each copula, the cumulative distribution function for the copulas can be derived as follows:

$$C^{Gu}(u, v) = \exp\left\{-((-\ln u)^{2.057} + (-\ln v)^{2.057})^{\frac{1}{2.057}}\right\}$$

$$C^{Cl}(u, v) = \left(u^{(-1)} + v^{(-1)} - 1\right)^{(1)}$$

The best-performing copula for catastrophic reinsurance contract evaluation, selected based on the lowest Akaike Information Criterion (AIC) value, is reported in **Table 4.**

Table 4. Copulas parameter estimation

Copula	AIC
Gumbel	-24.9581
Clayton	-2.5665

Gumbel copulas have the lowest AIC score compared to Clayton copulas. Hence, Gumbel copulas would be used in evaluating catastrophic reinsurance contracts. The joint cumulative distribution function for x and y will be constructed by these copulas as follows:

$$C^{Gu}(u,v) = \exp\left\{-((-\ln u)^{2.057} + (-\ln v)^{2.057})^{\frac{1}{2.057}}\right\}.$$

3.5 Monte Carlo Simulation

Using the estimated λ value, a Monte Carlo Simulation was conducted with 10000 iterations to model the possible variation in the number of earthquakes per year. The results of the simulations are shown in Table 5.

Table 5. Summary of Monte Carlo Simulation

Monte Carlo Simulation Descriptive	
Mean	1.2518
Standard Deviation	1.1173
Minimum	0
Maximum	7
1st Quartile (Q1)	0
Median	1
3rd Quartile (Q3)	2
P(earthquakes > 5/year)	0.2600

The histogram of the simulation shows that most years have an earthquake count centred around the average value of λ. The median value closely matches the mean, suggesting the distribution is nearly symmetric and consistent with the theoretical properties of the Poisson distribution.

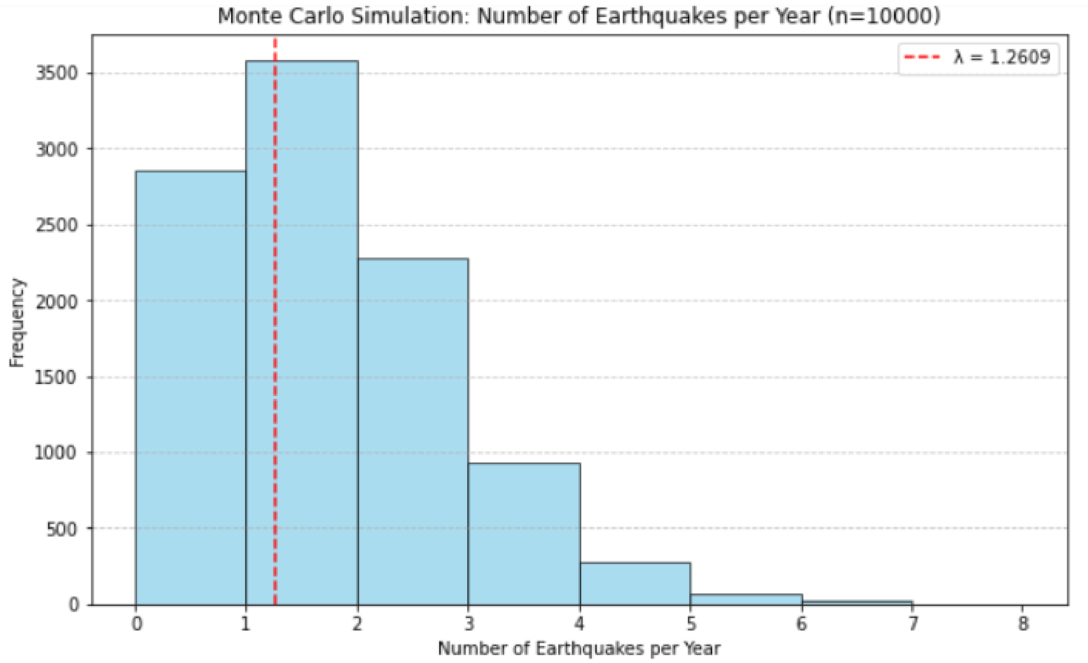

Figure 7. Monte Carlo Simulation: Number of Earthquakes per Year

The simulation supports the conclusion that the Poisson distribution is a suitable model to provide forecasting for future yearly occurrence of earthquakes and can be a useful tool for risk assessment if necessary.

Another Monte Carlo Simulation was conducted to generate 8378 potential joint outcomes ($\boldsymbol{X_i}$, $\mathbf{Y_i}$) of deaths ($\boldsymbol{X}$) and injuries ($\boldsymbol{Y}$) for extreme events which exceed the thresholds ($\boldsymbol{m_x = 517}$, $\boldsymbol{m_y = 1285}$). This simulated data of extreme deaths and injuries is then used in pricing. A Gumbel copula ($\boldsymbol{\theta = 2.057}$) is used within the simulation to model the dependence alongside Generalized Pareto Distributions with parameters ($\xi_x = \mathbf{0.7073562}$, $\sigma_x = \mathbf{447.822}$, $\xi_y = \mathbf{0.0071078}$, $\sigma_y = \mathbf{3816.675}$) for the marginals of the exceedances. The results of the simulation are plotted in Figure 8.

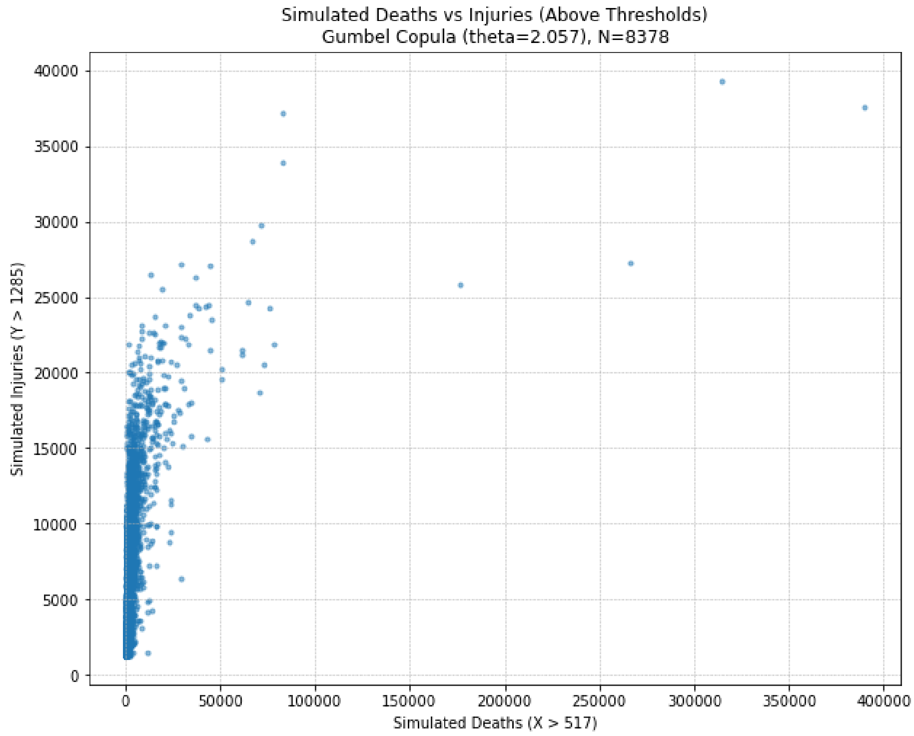

Figure 8. Scatterplot of Simulated Extreme Deaths and Injuries

It is evident in the scatterplot above that there is a clear positive association between the simulated extreme deaths and injuries beyond the threshold. In other words, events that result in a higher number of deaths tend to also result in higher number of injuries. This pattern is consistent with the dependence structure imposed by the Gumbel copula, which is specifically suited for modelling upper tail dependence.

3.6 Case Implementation and Expected Reinsurance Value Calculation

This section details the practical implementation of our model for valuing a catastrophe reinsurance contract in a real-world scenario. The valuation and premium calculation are conducted under the following key assumptions for Company A (the primary insurance company) and Company B (the reinsurance company):

a. Reinsurance Contract Structure:
   Company B provides stop-loss catastrophe reinsurance to Company A. This contract stipulates that Company B will cover any losses exceeding Company A's annual retention limit of Rp15,000,000,000 (fifteen billion Indonesian Rupiah) per catastrophic event within a one-year period.
b. Average Claim Payouts per Person:
   1) For life insurance: Rp 500,000,000 per death.
   2) For health insurance: Rp 15,000,000 per injury.
c. Company A's Insurance Penetration:
   These percentages represent the proportion of the Indonesian population holding policies with Company A:
   1) For life insurance: 5% of the Indonesian population.
   2) For health insurance: 7% of the Indonesian population.
d. Discount Rate (i):
   A discount rate of 5.75%, is applied, based on the latest Bank Indonesia (BI) rate in 2025.

Based on these penetration rates and average payouts, the effective claim cost per affected individual for Company A, considering only its insured population, is calculated as follows:

$$C_1 = 5\,\% \times Rp\,500{,}000{,}000 = Rp\,25{,}000{,}000$$
$$C_2 = 7\,\% \times Rp\,15{,}000{,}000 = Rp\,1{,}050{,}000$$

where:

a. C_1= Represents the average claim cost per fatality for Company A's life insurance policies.
b. C_2= Represents the average claim cost per injury for Company A's health insurance policies.

Next, the asset pricing formula for the reinsurance premium is applied to the simulated data generated by the Monte Carlo process. This formula calculates the expected present value of the claims that exceed the retention limit. Specifically, the formula is:

$$\pi = \sum_{t=1}^{N} f(X_t\,,\,Y_t)(1 + i)^{-t}$$

where:

a. π = Total premium for the reinsurance contract
b. N = Number of simulations (10,000)
c. $f(X_t\,,\,Y_t)$ = Claims function, based on number of deaths (X_t) and number of injuries (Y_t) for each catastrophic event in simulation t. This function represents the actual loss to the primary insurer, Company A, for a given event, after considering the insurance penetration rates.
d. X_t = Represents the simulated number of deaths for a given catastrophic event t in the Monte Carlo simulation.
e. Y_i = Represents the simulated number of injuries for a given catastrophic event t in the Monte Carlo simulation.
f. i = Discount rate (5.75%)
g. t = Time Period (representing each simulated event within the total N simulations).

By incorporating the calculated effective claim costs ($\boldsymbol{C_1}$ and $\boldsymbol{C_2}$) and the specified retention limit, the claims function $\boldsymbol{f(X_t\,,\,Y_t)}$ can be defined. The total claim incurred by Company A for a single event before reinsurance recovery is $\boldsymbol{C_1X_t + C_2Y_t}$. The reinsurer (Company B) covers only the portion of this claim that exceeds Company A's retention limit of Rp15,000,000,000. This is expressed using the $\boldsymbol{max(0,\ldots)}$ function to ensure claims are non-negative:

$$\pi = \sum_{i=i}^{N} \max\{0, (25 \times 10^6\ X_i + 1{,}05 \times 10^4\ Y_i - 15 \times 10^9)\}\,(1 + 5.75\%)^{-ti}$$

Substituting the simulated data obtained from the 10,000 Monte Carlo iterations into this formula, the total premium for the reinsurance contract, which represents the single reinsurance premium Company A would pay to Company B, is calculated as:

$$\mathrm{P} = \frac{\sum_{i=1}^{10{,}000} \pi i}{10{,}000} = 17{,}395{,}932{,}553.808212$$

The calculated single reinsurance premium of Rp17,395,932,553.81 represents the actuarially sound price for transferring the defined catastrophic earthquake risk from Company A to Company B. This premium is the expected present value of the potential claims that Company B would be obligated to cover, i.e., those exceeding Company A's Rp15 billion retention limit for life and health insurance policies.

This premium value is highly plausible given the nature of catastrophe reinsurance and the inherent characteristics of earthquake risk in Indonesia. While this sum may appear substantial, it is commensurate with the potential for extremely large, albeit infrequent, losses that such events can trigger. Catastrophe reinsurance is specifically designed to protect primary insurers from tail risk – the low-probability, high-severity events that could otherwise lead to severe financial distress or even bankruptcy. The calculated premium reflects the cost of offloading this critical exposure.

To contextualize this premium, a comparison against a hypothetical total annual premium received by Company A for its life and health insurance products is conducted. Assuming Indonesia's population is approximately 280 million people:

a. Company A's life insurance policyholders: 5% of 280 million = 14 million people.
b. Company A's health insurance policyholders: 7% of 280 million = 19.6 million people.

Assuming, hypothetically, the average annual premium for a life insurance policy is Rp 1,000,000 and for a health insurance policy is Rp 500,000, then Company A's approximate total annual premium income from these products would be:

a. Life insurance premiums:
$$14{,}000{,}000\ policies \times Rp\ 1{,}000{,}000/policy = Rp\ 14{,}000{,}000{,}000{,}000$$
b. Health insurance premiums:
$$19{,}600{,}000\ policies \times Rp\ 500{,}000/policy = Rp\ 9{,}800{,}000{,}000{,}000$$
c. Total hypothetical annual premium income:
$$Rp\ 14{,}000{,}000{,}000{,}000 + Rp\ 9{,}800{,}000{,}000{,}000 = Rp\ 23{,}800{,}000{,}000{,}000$$

Comparing the calculated reinsurance premium of Rp17,395,932,553.81 to this hypothetical total annual premium income (Rp 23.8 trillion), the reinsurance premium represents a small fraction (approximately 0.073%). This comparison underscores that the reinsurance premium is not intended to cover all claims but specifically acts as a crucial safeguard against the financial consequences of infrequent, highly impactful catastrophic events that exceed the primary insurer's capacity to absorb. This relatively small cost, in proportion to total premium income, provides significant financial stability and resilience for Company A, ensuring it can fulfill its obligations to policyholders even in the direst disaster scenarios. It facilitates the effective transfer of severe tail risk, which is a core function of the reinsurance market.

4. CONCLUSIONS

This research analyzes Indonesian earthquake data from 1979 to 2025, focusing on events that affected between 1 and 50 people to avoid extreme outliers (like the 2004 Aceh earthquake). The data reveals that while most earthquakes cause relatively few casualties, a few rare but devastating events lead to thousands of deaths and injuries, resulting in a heavily right-skewed distribution. This extreme variability highlights the importance of using advanced statistical methods, such as extreme value theory, to model these risks accurately. The study estimates that earthquakes occur at an average rate of about 1.26 per year, which follows a Poisson distribution. For extreme events, by setting thresholds of 517 deaths or 1,285 injuries, the Generalized Pareto Distribution (GPD) was applied to model the distribution. Furthermore, since there is a strong dependence between deaths and injuries in major disasters, Gumbel copula is used to model this dependency.

Using Monte Carlo simulations, the research generated ten thousand potential disaster scenarios to estimate expected reinsurance costs. Assuming a retention limit of Rp15 billion for the insurer and fixed claim values per affected policyholder, the model calculated the expected reinsurance payout. The results help insurers and reinsurers better prepare for catastrophic events by pricing coverage more accurately. Specifically, the calculated single reinsurance premium is Rp17,395,932,553.81. This value represents the actuarially sound price for transferring the defined catastrophic earthquake risk (exceeding the Rp15 billion retention limit) from the primary insurer to the reinsurer, providing a concrete cost for this critical risk transfer. While substantial, this premium is plausible given the high-severity, low-frequency nature of the events it covers, protecting against significant financial distress or potential bankruptcy from tail risks. Contextualizing this, the reinsurance premium typically represents a small fraction of an insurer's total annual premium income from covered life and health insurance products, underscoring its role as a vital safeguard for financial stability rather than a cost covering all claims.

However, the study has some limitations. Excluding the 2004 mega-disaster may underestimate worst-case scenarios, and fixed assumptions about insurance coverage and claim values might not reflect real-world variability. Future research could expand the dataset, test alternative copula models, and explore dynamic insurance penetration rates for more precise risk assessment. Overall, this approach provides a valuable framework for managing earthquake-related risks in Indonesia's reinsurance market.

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