IAJ
Indonesian Actuarial Journal
Persatuan Aktuaris Indonesia (PAI)

Vol. 01, No. 01, July (2025)
e-ISSN: xxxx-xxxx
https://ojs.aktuaris.or.id/iaj/index

Forecasting Indonesian Rupiah-to-US Dollar Exchange Rate 2020-2025 Using a Fuzzy Time Series Markov Chain Model

Tiur Masayu Rahmadani [1], Rosa Maeni Mutianti [2*], Camelia Miftahur Rizki Kiem Hwa [3], Iftha Nikmatul Khasanah [4], Winner Yeo [5], Kgs. M. Rifat Alfarisi [6], Yohana Joevanca Kurniawan [7], Adriano Fadlan Juwono [8], Mey Lista Tauryawati [9]

[1,2,3,4,5,6,7,8,9] *Business Mathematics, Universitas Prasetiya Mulya, Tangerang, Indonesia*

**Corresponding email:* maenirosa6@gmail.com

Abstract

The exchange rate of the Indonesian Rupiah against the US Dollar experiences frequent fluctuations, making economic forecasting and financial planning more difficult. This study aims to enhance exchange rate prediction accuracy by combining Fuzzy Time Series with Markov Chain probability transitions. The approach is grounded in the idea that probabilistic modeling of state changes improves the representation of dynamic currency behavior. Using daily IDR/USD data from April 2020 to March 2025, the methodology involves two main steps: fuzzifying historical exchange rate data into linguistic variables, and applying a Markov Chain to compute transition probabilities between these fuzzy states. The model's forecasting accuracy is evaluated using mean absolute percentage error. Results show that the hybrid model achieves a lower error rate of 0.50%, compared to 0.61% using conventional Fuzzy Time Series alone. This demonstrates the hybrid model's ability to capture both sudden market changes and stable patterns effectively. The findings suggest that the integration of Markov Chain transitions significantly improves the predictive performance of fuzzy-based models. In conclusion, this hybrid method provides a practical and reliable forecasting tool for financial analysts and policymakers. Future research could include additional economic indicators and explore alternative probability weighting methods to further enhance model accuracy.

Article History:
Received: 5 May 2025
First Revised: 8 May 2025
Accepted:15 July 2025
Published: 31 July 2025

Keywords:
Exchange Rate, Forecasting, Fuzzy Time Series, Hybrid Model, Markov Chain

1. INTRODUCTION

The exchange rate between the Indonesian Rupiah (IDR) and the US Dollar (USD) plays a critical role in Indonesia's economic stability, influencing trade, investment, and monetary policy [1]. However, exchange rate fluctuations are inherently volatile, driven by complex factors such as geopolitical events, market sentiment, and macroeconomic indicators. Accurate forecasting of these fluctuations is essential for policymakers, businesses, and investors to mitigate risks and make informed decisions. Traditional time series models often struggle to capture the uncertainty and nonlinearity of exchange rate movements [2], necessitating more advanced methodologies.

Fuzzy Time Series (FTS) has emerged as a powerful tool for handling imprecise and incomplete data in forecasting problems [3]. Unlike conventional statistical methods, FTS models linguistic variables and fuzzy logic to represent historical patterns, making them particularly suited for financial time series with high volatility. However, standard FTS models may lack adaptability to dynamic transitions between states [4]. To address this limitation, this study integrates Markov Chains into the FTS framework, enhancing its ability to model probabilistic transitions between fuzzy states and improve prediction accuracy [5].

The primary objective of this research is to develop a Fuzzy Time Series-Markov Chain (FTS-MC) hybrid model for forecasting the IDR/USD exchange rate. The study utilizes historical daily exchange rate data from April 2020 to March 2025, sourced from Investing.com, and evaluates the model's performance using the Mean Absolute Percentage Error (MAPE) [6]. By comparing the FTS-MC model with a baseline FTS model, this study demonstrates the superiority of the hybrid approach in capturing exchange rate dynamics. The results reveal that the FTS-MC model achieves a MAPE of 0.50%, outperforming the standalone FTS model (0.61%), thus validating its efficacy for high-accuracy forecasting.

This study contributes to the field of financial forecasting by proposing a robust hybrid model that combines the interpretability of FTS with the dynamic adaptability of Markov Chains. The findings not only highlight the model's practical utility for exchange rate prediction but also pave the way for future applications in other volatile financial markets [5], [7]. The remainder of this paper is organized as follows: Section 2 details the methodology, including data preprocessing and the FTS-MC algorithm. Section 3 presents empirical results and comparative analyses, while Section 4 discusses implications, limitations, and directions for further research.

2. METHODS

This study employs a quantitative approach with a descriptive-predictive method, aiming to analyze historical patterns of the Indonesian Rupiah (IDR) exchange rate against the US Dollar (USD) and forecast future values. The forecasting model is based on the Fuzzy Time Series (FTS) method, enhanced with a Markov Chain to improve predictive accuracy. The research utilizes secondary data comprising daily closing exchange rates of the Rupiah to the US Dollar, spanning from April 1st 2020 to March 31th 2025. The data were sourced from the official website Investing.com and downloaded in Excel format using a documentation method. Prior to analysis, the dataset underwent a cleaning process to remove inconsistencies and outliers, ensuring its readiness for predictive modeling.

Data were then analyzed numerically and structured into a forecasting model by applying FTS logic. This technique leverages linguistic variables to handle uncertainty and imprecision inherent in time series data. The integration of the Markov Chain enables the calculation of transition probabilities between fuzzy states, increasing the model's ability to adapt to dynamic exchange rate movements. This concept was first applied by Tsaur to analyze the prediction accuracy of the Taiwanese Dollar's exchange rate against the US Dollar [5]. The testing process begins by determining the value of parameter D and its variations in the initial stage of forming the universe of discourse U. After the Fuzzy Time Series (FTS) Markov Chain model is constructed based on historical exchange rate data, several testing scenarios are conducted to evaluate the accuracy level of each model.

Accuracy measurement is performed using the Mean Absolute Percentage Error (MAPE) method. From the test results, the FTS Markov Chain model with the best accuracy level will be selected. Subsequently, this best model will be compared with an FTS model without the Markov Chain combination to see the extent of the prediction performance improvement. The selected FTS Markov Chain model will then be used to predict the Rupiah's exchange rate against the US Dollar for the subsequent time period.

3. RESULT AND DISCUSSION

The data used in this study is historical exchange rate data containing information such as date and closing price (Last). The data consists of 1,261 entries covering the period from April 1, 2020 to April 14, 2025.

Figure 1. Daily Closing Price Movement (Last) from 2020 to 2025

As an initial step in data analysis, an identification of extreme values is carried out in the "Last" column which represents the closing price of the rupiah exchange rate in each observation period. The purpose of this stage is to obtain a general picture of the range of price fluctuations that occurred during the observation period. Based on the results of data processing, the minimum value is obtained (D_{min}) which is 13,870 on April 1st, 2020 and the maximum value (D_{max}) which is 16,860 on March 29th 2024. Based on the values of $D < min >$ and $D < max >$, the value of d_1 dan d_2 can be determined. The values of d_1 dan d_2 are positive integers which are determined by the researcher as a multiplier factor for the data range to expand the boundaries of the universe of discourse. In this study, the selected value are $d_1 = 0.1$ and $d_2 = 0.3$. The values of $D < min >$, $D < max >$, D_1 and D_2 are used to form the universe of discourse, U.

Table 1. Values of D for rupiah exchange rate

	D_{min}	D_{max}	d_1	d_2	Model
Exchange Rate	13,870	16,860	0.1	0.3	Model 1

The experiment for the exchange rate of Model 1 using Fuzzy Time Series (FTS) Markov Chain with the following steps:

3.1. Determine the Universe of Discourse U Based on Historical Data in Table 1.

After the data cleaning process is complete, the next stage is to form the U universe as a basis for further analysis. The U universe is determined based on the minimum and maximum values of the data, namely. From the historical data, the smallest value (D_{min}) and the largest value (D_{max}) are obtained. The values d_1 and d_2 are randomly chosen positive integers that determine the effective interval length used to simplify the interval formation.

$$D_{min} = 13{,}870$$
$$D_{max} = 16{,}860$$
$$d_1 = 0.1$$
$$d_2 = 0.3$$

The Data Range is calculated as:

$$Data\ Range = D_{max} - D_{min} = 2{,}990$$

The universe of U is calculated based on the following formula:

$$U = [D_{min} - d_1(D_{max} - D_{min})\ ;\ D_{max} + d_2(D_{max} - D_{min})]$$

$$U = [13{,}870 - 0.1(2{,}990)\ ;\ 16{,}860 + 0.3(2{,}990)] = [13{,}571\ ;17{,}757]$$

The formation of this universe is important to define the boundaries of the scope of the analysis that are more flexible and comprehensive, and to be the basis for subsequent processes such as class determination, fuzzification, and stochastic system modeling.

The selection of d_1 and d_2 values is arbitrary but must be positive. In this case, we chose $d_1 = 0.1$ and $d_2 = 0.3$ to slightly expand the universe range beyond the actual minimum and maximum values. Choosing small values close to 0 ensures that the universe remains close to the actual data range, minimizing the influence of outliers and preventing the inclusion of irrelevant data points. However, this slight extension also provides enough flexibility to accommodate uncertainty or future variations in the data, which is crucial in fuzzy or stochastic modeling contexts.

3.2. Determine the number of intervals and the length of each interval

a. Determine the number of intervals
   Using sturges formula:

$$K = 1 + 3.322 \times log_{10}(n)$$

$$K = 1 + 3.322 \times 3.1005 \approx 1 + 10.2973 = 11.2973$$

$$K \approx 12$$

   Hence, the number of intervals K for $n = 1{,}261$ is 12, after being rounded up (as per the general rounding convention in this context).

b. Determine the length of the intervals
   Using the formula:

$$l = \frac{(D_{max} + d_2(D_{\ max} - D_{min})) - (D_{min} - d_1(D_{\ max} - D_{min})}{K}$$

$$l = \frac{17{,}757 - 13{,}571}{12} = 348.8333$$

   The length of each interval is about 348.84 units. With this length, the universe of discourse U can be divided evenly into 12 classes/intervals.

Table 2. Values of u1-u12 rupiah exchange rate

No.	Un	Value
1	u1	(13571.0000, 13919.8333)
2	u2	(13919.8333, 14268.6667)
3	u3	(14268.6667, 14617.5000)
4	u4	(14617.5000, 14966.3333)
5	u5	(14966.3333, 15315.1667)
6	u6	(15315.1667, 15664.0000)
7	u7	(15664.0000, 16012.8333)

8	u8	(16012.8333, 16361.6667)
9	u9	(16361.6667, 16710.5000)
10	u10	(16710.5000, 17059.3333)
11	u11	(17059.3333, 17408.1667)
12	u12	(17408.1667, 17757.0000)

Every $u1$ up until $u12$ is a fuzzy set or value cluster formed from numeric data based on a certain range. In the stochastic fuzzification process, numeric data is converted into a fuzzy representation by dividing the value domain into predetermined intervals. The input data is then mapped into one of these fuzzy sets according to the value it has, depending on which range the value is in. It is called stochastic because the formation of the range or interval generally involves a statistical approach, such as dividing the data domain based on a uniform probability distribution or following the actual value distribution pattern in the data. This approach allows flexible and adaptive data grouping to the statistical characteristics of the data being analyzed. Find the middle value of each class. The middle value of the rupiah exchange rate is presented in Table 3.

Table 3. The value of the rupiah exchange rate

No	mi	Value
1	m1	13745.4167
2	m2	14094.2500
3	m3	14443.0833
4	m4	14791.9167
5	m5	15140.7500
6	m6	15489.5833
7	m7	15838.4167
8	m8	16187.2500
9	m9	16536.0833
10	m10	16884.9167
11	m11	17233.7500
12	m12	17582.5833

The middle value of each fuzzy set $m1$ up until $m12$ shows the central numerical representation of each fuzzy interval that has been formed through the stochastic fuzzification process. This value is calculated from the average of the lower and upper limits of each interval, and serves as a reference point or typical representation of each cluster. For example, the middle value $m1$ is 13745.4167 which comes from the range

[13571.0000, 13919.8333], while the middle value $m12$ is 17582.5833 from the range [17408.1667, 17757.0000].

These midpoints are used as part of the process of transforming numeric data into fuzzy form, which can then be used in fuzzy logic-based analysis or in developing predictive systems. Once the data is mapped into one of the 12 fuzzy intervals, the midpoints serve as fixed representations that replace the original data in subsequent calculations. This approach keeps the data structure in a more flexible form and is able to capture the uncertainty and natural variation in numeric data. These midpoints also facilitate the defuzzification process at the final stage of fuzzy analysis, since each fuzzy output can be returned to a meaningful numeric form using the midpoints of the associated clusters.

3.3 Dividing Fuzzy Sets for the Universe of Discourse U

The fuzzy set for each data is as follows.

$$A1 = \left\{\frac{1}{u1} + \frac{0.5}{u2} + \frac{0}{u3} + \frac{0}{u4} + \frac{0}{u5} + \frac{0}{u6} + \frac{0}{u7} + \frac{0}{u8} + \frac{0}{u9} + \frac{0}{u10} + \frac{0}{u11} + \frac{0}{u12}\right\}$$

$$A12 = \left\{\frac{0}{u1} + \frac{0.5}{u2} + \frac{1}{u3} + \frac{0.5}{u4} + \frac{0}{u5} + \frac{0}{u6} + \frac{0}{u7} + \frac{0}{u8} + \frac{0}{u9} + \frac{0}{u10} + \frac{0.5}{u11} + \frac{1}{u12}\right\}$$

The following hows the fuzzy expression for each fuzzified data, denoted as $A1$ up until $A12$. Every expression from $A1$ up until $A12$ states a linear combination of the degree of membership to the fuzzy sets $u1$ to $u12$, with a certain weight (1, 0.5, or 0). This expression shows that one data is not only a member of one fuzzy set absolutely, but can have a degree of membership in several fuzzy sets at once—a key concept in fuzzy logic.

For example, the expression

$$A1 = \left\{\frac{1}{u1} + \frac{0.5}{u2} + \frac{0}{u3} + \frac{0}{u4} + \frac{0}{u5} + \frac{0}{u6} + \frac{0}{u7} + \frac{0}{u8} + \frac{0}{u9} + \frac{0}{u10} + \frac{0}{u11} + \frac{0}{u12}\right\}$$

means that the first data ($A1$) completely become a member of a fuzzy set. For example, the expression: $u1$ (membership value 1), partial (0.5) becomes a member of $u2$, and is not a member of any other set (value 0). Any other expression ($A2$up until $A12$) follows a similar pattern, but with a shift to the right, reflecting a moving window process or the spread of membership across fuzzy intervals. This is consistent with the triangular or trapezoidal fuzzification approach, where a data set can belong to two or three fuzzy sets with varying degrees of membership, depending on its distance from the midpoint of each interval. This step is an important stage in the fuzzy inference process, because the results of this fuzzy expression will later be used to form fuzzy rules or make predictions based on the fuzzy data that has been formed. This process also refines the data representation so that the system can handle the uncertainty and imprecision that are common in real-world data.

3.4. Fuzzification of Historical Data

Fuzzification of the rupiah exchange rate can be seen in Table 4.

Table 4. Fuzzification of the rupiah exchange rate

No	Actual Data	Fuzzy Data of Rupiah Exchange Rate
1	16786.6	A10
2	16783.3	A10

. . .	. . .	. . .
1260	16465.0	A9
1261	16450.0	A9

This stage shows the fuzzification results of historical data which is an important part in transforming numeric data into fuzzy form. The "Actual Data" column contains the actual values of historical data, while the "Rupiah Exchange Rate Fuzzy Data" column shows the fuzzy set ($A1$ up until $A12$) which represents each value based on a previously determined range of fuzzy values. By performing this fuzzification, the original numerical data is converted into linguistic or symbolic form ($A1, A2, \ldots\ldots, A12$), which can be used to form patterns, fuzzy rules, or predictive models based on fuzzy logic. This stage is important in fuzzy systems because it allows decision making that takes into account uncertainty and tolerance for ambiguity in the data.

3.5. Fuzzy Logical Relationship (FLR)

Fuzzy Logical Relationship (FLR) is an important part of the Fuzzy Time Series method which is used to understand the relationship patterns between data based on fuzzy sets. In determining FLR, the first step is to perform fuzzification, which is to change the values of numeric data into linguistic categories (e.g. A1, A2, ..., An) based on a certain range of values. After the data is fuzzified, the relationship between times is observed, for example from April 14, 2025 to April 13, 2025, if both values are in the same fuzzy set, namely A10, then FLR is formed: A10 → A10. This relationship describes the transition from one condition to the next in the form of a fuzzy set. FLR is very useful for identifying historical patterns and is the basis for the forecasting process using a fuzzy approach. By analyzing time series based on FLR, we can build a prediction model that takes into account uncertainty and ambiguity in historical data, especially in data that is fluctuating or nonlinear.

3.6. Determining Fuzzy Logical Relationship Group (FLRG)

Table 5. Fuzzy Logical Relationship Group from the rupiah exchange rate

No	Current State	Next State
1	A1	3(A2), 5(A1)
2	A10	5(A10),1(A9)
3	A2	11(A3),150(A2),3(A1)
4	A3	240(A3),7(A4),11(A2)
5	A4	168(A4),10(A5),7(A3)
6	A5	119(A5),7(A6),10(A4)
7	A6	196(A6),15(A7),7(A5)
8	A7	102(A7),15(A6),3(A8)
9	A8	8(A9),113(A8),3(A7)
10	A9	33(A9),8(A8)

In the context of the Fuzzy Time Series (FTS) method, Fuzzy Logical Relationship Group (FLRG) is a grouping of logical relationships (Fuzzy Logical Relationship/FLR) that have the same initial state (current state). After the FLR is formed, the FLRG is arranged by grouping all next states (destination states) that emerge from one current state. This aims to capture all possible transitions that may occur from a particular fuzzy condition, along with their frequencies, so that this approach is relevant for processing time series data that is stochastic or contains elements of uncertainty. For example, from the data displayed, the current state A6 has a FLRG of 196(A6),15(A7),7(A5), which means that from condition A6, the transition to A6 occurs 196 times, to A7 15 times, and to A5 7 times. In other words, the greatest possibility of transition A6 is to remain in A6, but it does not rule out the possibility of moving to A7 or A5. This FLRG is an important basis for making stochastic predictions using frequency weights or transition probabilities to calculate future prediction values more realistically and accurately.

3.7. Creating a Transition Probability Matrix

Table 6. Transition Probability Matrix

	A1	**A10**	**A2**	**A3**	**A4**	**A5**	**A6**	**A7**	**A8**	**A9**
A1	0.6250	0.0000	0.3750	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A10	0.0000	0.8333	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1667
A2	0.0183	0.0000	0.9146	0.0671	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A3	0.0000	0.0000	0.0426	0.9302	0.0271	0.0000	0.0000	0.0000	0.0000	0.0000
A4	0.0000	0.0000	0.0000	0.0378	0.9081	0.0541	0.0000	0.0000	0.0000	0.0000
A5	0.0000	0.0000	0.0000	0.0000	0.0735	0.8750	0.0515	0.0000	0.0000	0.0000
A6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0321	0.8991	0.0688	0.0000	0.0000
A7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250	0.8500	0.0250	0.0000
A8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0242	0.9113	0.0645
A9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1951	0.8049

The Markov Transition Probability Matrix (R) represents the probability of moving from one fuzzy state to another based on the transition frequencies in the FLRG. Each entry R_{ij} represents the probability of transitioning from state A_i to A_j, calculated as the ratio of the transition frequency to the total transitions from A_i. This matrix adds a stochastic element to the Fuzzy Time Series model, allowing for more accurate predictions of random or uncertain data. By multiplying the current state vector by this matrix, an estimate of the probabilistic distribution for the next time can be obtained.

3.8. Determining Initial Forecast Values

After determining the initial forecast values, the results obtained are summarized in Table 7.

Table 7. Initial Forecast Values

No.	**Date**	**Last**	**Fuzzy Sets**	$F(t)$
1	14/04/2025	16786.6	A10	NaN
2	13/04/2025	16783.3	A10	16826.7778

. . .	. . .	. . .	. . .	. . .
1260	2/4/2020	16475.0	A9	16468.0183
1261	1/4/2020	16450.0	A9	16468.0183

The determination of the initial forecast value is done by combining the fuzzification results and the transition probabilities between fuzzy sets. At this stage, each historical data is classified into a certain fuzzy set (e.g. A10, A9, etc.), then the Markov transition matrix is used to estimate the future value of F(t). The value of F(t) is obtained from a weighted average based on the transition probabilities from the current state to another state. This result is the basis for forecasting in the Stochastic Fuzzy Time Series model, which captures the uncertainty in time series data more realistically and accurately.

3.9. Calculating the Adjustment Value on the Forecast Results

The adjustment values on the forecast results are summarized in Table 8.

Table 8. Forecast Adjustment Value Results

No.	**Date**	**Last**	***Fuzzy Set***	$F(t)$	Adjustment
1	14/4/2025	16786.6	A10	NaN	0.00
2	13/4/2025	16783.3	A10	16826.777778	0.00
. .					
1257	7/4/2020	16175.0	A8	16201.315860	0.00
1258	6/4/2020	16412.5	A9	16201.315860	174.42
1259	3/4/2020	16425.0	A9	16468.018293	0.00
1260	2/4//2020	16475.0	A9	16468.018293	0.00
1261	1/4/2020	16450.0	A9	16468.018293	0.00

The "Adjustment" column in the table reflects corrections to the predicted results based on the transition between fuzzy sets from one time period to the next in the Fuzzy Time Series (FTS) method. If there is no change between fuzzy sets, the adjustment is zero. However, when a shift occurs, either upward or downward, the adjustment is calculated by multiplying the number of transition steps by half of the fuzzy interval length $(\frac{1}{2})$. This adjustment helps improve predictions to better match actual values by considering the direction and magnitude of trend changes, making the FTS model more adaptive and responsive to time dynamics.

3.10. Calculate the Forecast Results Adjusted by the Equation

Table 9. Forecasting

No.	Date	Last	Fuzzy Set	$F(t)$	Adjustment	Adjusted $F(t)$
1	14/4/2025	16786.6	A10	NaN	0.00	NaN
2	13/4/2025	16783.3	A10	16826.7778	0.00	16826.7778
. . .						
1261	1/4/2020	16450.0	A9	16468.0183	0.00	16468.0183

The forecasting results presented demonstrate the application of the stochastic approach in the fuzzy time series method, where each forecast value not only depends on the deterministic pattern but also considers the random component. This can be seen from the presence of an adjustment column with varying values, such as 0.00 or 174.42, which adds a stochastic factor to the initial forecast results. For example, in the data dated 6/4/2020, the forecast value was adjusted from 16201.32 to 16375.74 after adding a stochastic component of 174.42, showing the flexibility of the model in capturing random fluctuations. Grouping data into fuzzy sets (A8, A9, A10) also reflects stochastic variations in data patterns, where each set represents a range of values with a certain degree of uncertainty. As an initial step in comparing model performance, Fuzzy Time Series (FTS) is built and used as a baseline. In this FTS model, the forecasting results for each period are recorded as "F(t)". However, this FTS model only considers the relationship between data in a simple form without considering the transition probability between statuses, so that the sensitivity to changes in trend patterns is more limited. As a continuation, the FTS model is developed into FTS-MC by adding a prediction adjustment mechanism through Markov Chain. In the FTS-MC model, the forecasting results are adjusted based on the transition probability between previously created fuzzy sets, resulting in a new prediction value recorded as "Adjusted F(t)".

3.11. Accuracy Comparison

Table 10. MAPE Value

Method	MAPE Value
Fuzzy Time Series	0.61%
Fuzzy Time Series-Markov Chain	0.50%

After the Fuzzy Time Series model was created for the comparison of Fuzzy Time Series with Markov Chain, the performance evaluation of the two models was carried out using Mean Absolute Percentage Error (MAPE) as a measure of accuracy. Based on the evaluation results, the MAPE value for Fuzzy Time Series (FTS) was 0.61% and Fuzzy Time Series-Markov Chain (FTS-MC) 0.50%. The values obtained by both methods indicate that both are able to provide forecasting results with a very low error rate, which means that the predictions produced are very close to the actual value. Because the MAPE value produced by FTS-MC is smaller than FTS, this indicates that the addition of the Markov Chain component can increase accuracy by calculating the transition between exchange rate statuses more dynamically. The decrease in MAPE value of 0.11% from FTS to FTS-MC, although it seems small, indicates an increase in the stability and sensitivity of the model to exchange rate change patterns.

3.12. Forecasting Results with Fuzzy Time Series (FTS) Markov Chain

The forecasting results are compared with actual data throughout 2020 to 2025 to evaluate the model performance. In 2020, the movement of the Rupiah exchange rate against the Dollar showed a fairly sharp decline, especially in the second quarter, along with global uncertainty due to the Covid-19 pandemic. The Fuzzy Time Series-Markov Chain (FTS-MC) model is able to capture this decline pattern quite well. However, there are minor deviations at several data points, especially when there are sudden price fluctuations. In general, the model follows the direction of the exchange rate movement with acceptable accuracy, although the model's response to sudden changes still needs improvement.

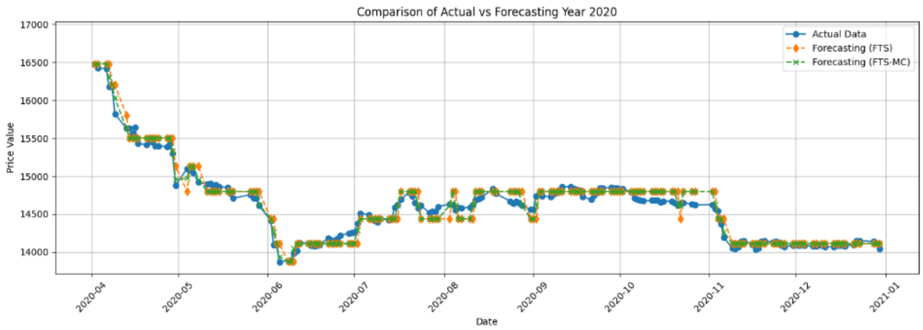

Figure 2. Graph of FTS 2020 and FTS-MC 2020 forecast results against actual data

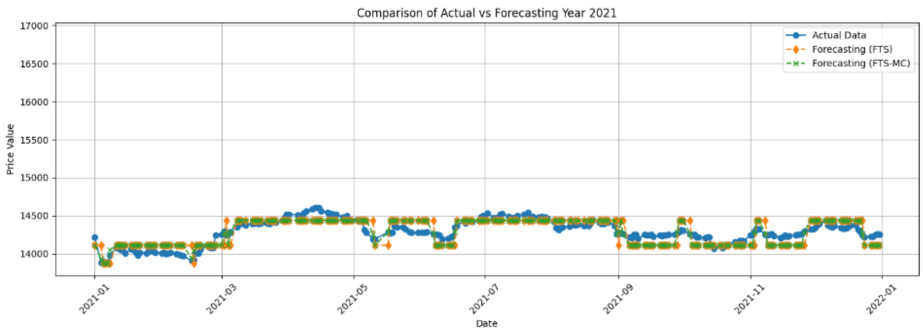

Figure 3. Graph of FTS 2021 and FTS-MC 2021 forecast results against actual data

Entering 2021, overall exchange rate volatility tends to decrease compared to the previous year. Price fluctuations have become more stable, so the FTS-MC model shows better performance. The forecast line is almost always close to the actual data, indicating the model's ability to capture medium- and short-term patterns with greater precision. However, small differences still appear when there are daily price changes that are random.

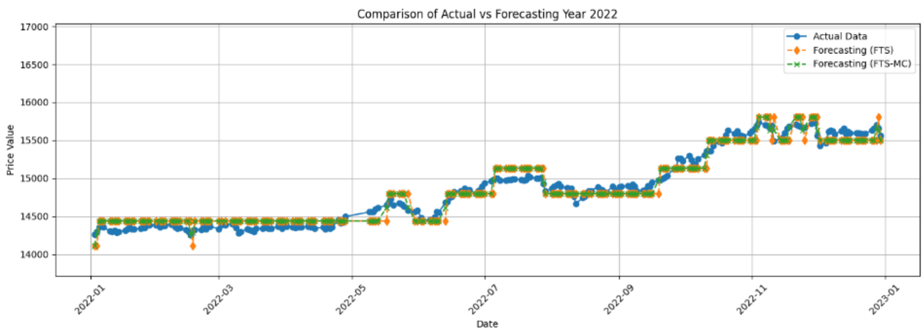

Figure 4. Graph of FTS 2022 and FTS-MC 2022 forecast results against actual data

In 2022, the exchange rate experienced a gradual upward trend, especially entering the second half of the year. The FTS-MC model was quite successful in following this upward trend, although there was a slight delay in capturing the beginning of the trend change. The difference between the forecast results and the actual data increased slightly compared to the previous year, but the general pattern is still well represented by the model.

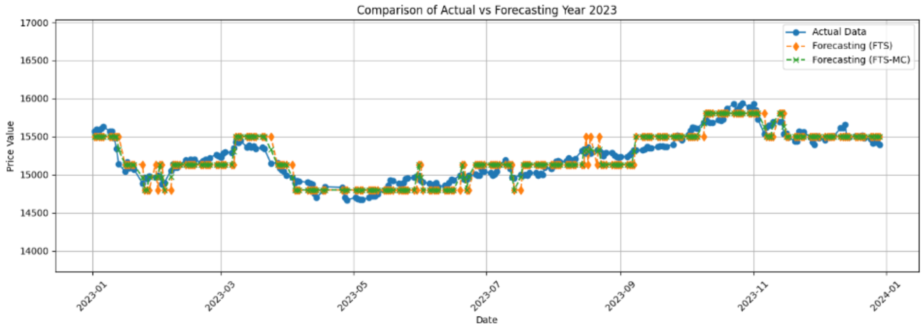

Figure 5. Graph of FTS 2023 and FTS-MC 2023 forecast results against actual data

The year 2023 saw higher volatility compared to previous years. There were several periods of rapid price declines and increases. The FTS-MC model was still able to follow the main trend of exchange rate movements, but its accuracy declined slightly, especially when there were quite extreme short-term fluctuations. This shows that the model's sensitivity to rapid price changes still needs to be improved.

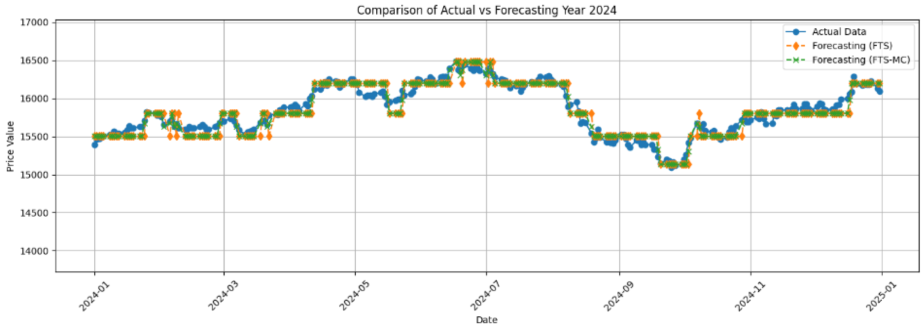

Figure 6. Graph of FTS 2024 and FTS-MC 2024 forecast results against actual data

During 2024, the exchange rate movement is more dynamic, with many trend changes in a relatively short period of time. Nevertheless, the forecast results still show a tendency close to the actual data. At certain phases, such as when there is a sharp spike or sudden drop, the model shows a delay in response, but overall, the medium-term trend is still well captured.

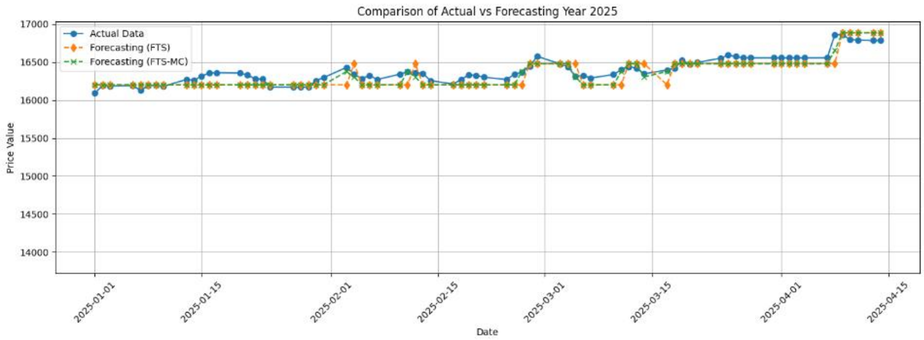

Figure 7. Graph of FTS 2025 and FTS-MC 2025 forecast results against actual data

In early 2025 to mid-April, the exchange rate movement showed a more stable tendency at high numbers. This condition makes the performance of the FTS-MC model look very good, with forecast results that are almost always close to the actual value. The horizontal pattern that occurs makes it easier for the model to maintain the accuracy of its predictions, indicating that FTS-MC is very effective in predicting stable market conditions. Based on the analysis results from 2020 to 2025, the Fuzzy Time Series-Markov Chain (FTS-MC) method has proven to be able to forecast the Rupiah exchange rate against the Dollar with quite high accuracy, especially in identifying medium and long-term trend patterns. The discrepancies that arise generally occur at moments with high volatility or very rapid trend changes.

4. CONCLUSIONS

This study successfully developed a Fuzzy Time Series-Markov Chain (FTS-MC) hybrid model to forecast the Indonesian Rupiah (IDR) exchange rate against the US Dollar (USD). By integrating Markov Chains into the Fuzzy Time Series framework, the model effectively captured the probabilistic transitions between fuzzy states, enhancing its adaptability to dynamic exchange rate movements. The FTS-MC model achieved a Mean Absolute Percentage Error (MAPE) of 0.50%, outperforming the standalone FTS model (MAPE: 0.61%). This demonstrates that incorporating Markov Chains significantly refines prediction accuracy by accounting for stochastic dependencies in exchange rate trends.The successful implementation of this model provides financial institutions and policymakers with a practical tool for exchange rate prediction. Future research could enhance the model's capabilities by incorporating macroeconomic indicators and testing its performance during different economic cycles.

5. REFERENCES

[1] C. Engel, "Exchange Rates and Interest Parity," in Handbook of International Economics, vol. 4, G. Gopinath, E. Helpman, and K. Rogoff, Eds. Elsevier, 2016, pp. 453–522.
[2] R. J. Hyndman and A. B. Koehler, "Another Look at Measures of Forecast Accuracy," Int. J. Forecast., vol. 22, no. 4, pp. 679–688, 2006.
[3] Q. Song and B. S. Chissom, "Fuzzy Time Series and Its Models," Fuzzy Sets Syst., vol. 54, no. 3, pp. 269–277, 1993.
[4] S. M. Chen and K. Tanuwijaya, "Multivariate Fuzzy Time Series for Forecasting Exchange Rates," Expert Syst. Appl., vol. 38, no. 8, pp. 10394–10402, 2011.
[5] R. C. Tsaur, "A Fuzzy Time Series-Markov Chain Model with an Application to Forecast the Exchange Rate Between USD and TWD," Appl. Soft Comput., vol. 12, no. 1, pp. 433–441, 2012.
[6] Investing.com, "USD/IDR Historical Data," 2025. [Online]. Available: https://www.investing.com/
[7] J. Kacprzyk and M. Krawczak, "Hybrid Fuzzy and Stochastic Models for Forecasting Financial Time Series," IEEE Trans. Fuzzy Syst., vol. 28, no. 5, pp. 934–948, 2020.