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Modelling The Probability of Insurance Company's Bankruptcy Using Integro-Differential Equation and Its Simulation

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Abstract

Ruin is one of the risks that insurance companies may face. Therefore, modeling the probability of ruin is a crucial aspect of risk management. This study aims to model and analyze the probability of ruin in insurance companies using integro-differential equations, assuming that the claim size follows a mixtures of two exponential distributions. The research methods are literature study and numerical simulation with phyton. Numerical calculations were performed using Python programming, and the results are presented in tables and graphs to facilitate analysis. The model is applied to three different cases. The findings indicate that the probability of ruin is inversely proportional to the initial capital and premium loading, while it is directly proportional to the expected value of claims. In other words, the probability of ruin decreases significantly as the initial capital and premium loading increase, whereas it increases as the expected value of claims rises. Therefore, the greater the surplus of an insurance company, the lower the probability of ruin.

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1. INTRODUCTION

Insurance is an agreement by which an insured person, called policy holder, binds themselves to an insurer, which can be an insurance company. Most individuals use insurance service to transfer risks by paying a certain amount of premium periodically to the insurance company. In this agreement, the insurer receives an amount of premium from the insured, aiming to provide a compensation over the loss that the insured might suffer from because of an event or uncertain risk [1]. This premium becomes obligatory for the insured for the protection guarantee provided by the insurance company. When the insured risks occur, the policy holder reserves the right to submit a claim to the insurance company to get the compensation as per the terms that have been agreed upon in the policy.

This claim payment affects the insurance company's assets because the claim submitted by the policy holder should be paid from the company's assets. These assets come from the insurance company's initial capital and premium earnings. An insurance company's revenue depends on the company's assets minus accumulated claims paid to policy holders. Therefore, this revenue will keep on changing from time to time depending on the amount of premium received and the claims submitted. As for the company's initial capital, its value will remain the same at all time. If at a period the amount of initial capital and premiums received is less than the amount of claims paid, the insurance company would face a loss risk that may lead to bankruptcy [2].

In an insurance company, a surplus process constitutes the process of accumulating wealth received from initial capital and the amount of premiums received, then minus the total accumulated claims paid. A surplus with a positive value shows the company's capability to pay its entire claim obligations. Meanwhile, a surplus with a negative value indicates the company's inability to meet its claim obligations. This means that the company is under the bankruptcy risks. To address this risk, companies need to consider the possible ruin probability. The main challenge in this strategy is to find the right model to calculate the said ruin probability [3].

The previous conducted by Diba et al. [4] and Erizal and Septiadi [5] investigate the ruin insurance companies' probability modelling and simulation by analyzing the premium value and claim size following an exponential distribution. Dwiyawara and Widodo [6] study a ruin probability modelling and simulation using integro-differential equation. The three studies explore a insurance company's ruin probability modelling using different claim size assumptions. In Diba et al. [4] and Erizal and Septiadi [5] the claim size are assumed to be exponentially distributed, and in Dwiyawara and Widodo [6] the claim size is assumed to be normally distributed.

In the current research, the insurance company's ruin probability model will be discussed using an integro-differential equation. The claim size distribution used was a mixtures of two exponential distributions. Thus, the problems in this research were formulated as: how is the insurance company's bankruptcy probability model using integro-differential equation with a claim size distributed with two mixtures exponential distributions, and how is the result of analysis of insurance company's ruin probability model using integro-differential equation with a claim size distributed with two mixtures exponential distributions. Furthermore, this research aims at obtaining an insurance company's ruin probability model and analyzing its results based on integro-differential equation approach by assuming that the claim is distributed with two mixtures exponential distribution.

2. RESEARCH METHOD

The method used in this research was literature study by collection relevant information from various sources such as books, ebooks, and journals on the topic to be discussed and numerical simulation with phyton. The theories used included Poisson's process, claim process, compound Poisson surplus process, bankruptcy probability, adjustment coefficient, bankruptcy probability upper limit, premium loading, integro-differential equation, mixtures of two exponential distributions, and linear differential equation.

2.1 Proses Poisson Process

The stochastic process $\{N(t), t \geq 0\}$ is referred to as a counting process if $N(t)$ is the number of events occuring until the time t. The stochastic process $N(t)$ is a continuous-time stochastic process with the number of events calculated until the time t. One of the stochastic process that can be used to determine the number of events until the time t is the Poisson process. Therefore, $\{N(t), t \geq 0\}$ is a Poisson processes. [7].

2.2 Claim Process

Definition 1 [8].
The stochastic process $\{S(t), t \geq 0\}$ with $S(t) = X_1 + X_2 + X_3 + \cdots + X_{N(t)}$ is said to be a compound Poisson process if $\{N(t), t \geq 0\}$ is the Poisson process, and X_i ; $i = 1,2,3, \dots, N(t)$ is a random variable which is independent and identically distributed and not dependent on $\{N(t), t \geq 0\}$.

2.3 Compound Poisson Surplus Process

A surplus process at time t is defined as the insurance company's initial capital (u) plus the premium it received constantly per a time unit (ct) minus accumulated claims paid at time interval $[0, t]$, i.e. $S(t)$. Mathematically, the surplus process is expressed as:

$$U(t) = u + ct - S(t);\ \ u \geq 0, c > 0, t \geq 0. \tag{1}$$

In premium calculation, it is assumed that the premium is paid continuously and constantly per time unit. A net premium has a cost burden expressed as $ct > E[S(t)]$, thus $c > \lambda\mu$; where $\mu = E[X_i]$; $i = 1,2, \dots N(t)$

and λ is a parameter of the Poisson distribution. Therefore, equation (2) is obtained with $\theta > 0$ being referred to as premium loading [2]

$$c = (1 + \theta)\lambda\mu. \tag{2}$$

2.4 Ruin Probability

Ruin probability is defined as the probability for a bankruptcy to occur for the first time, i.e., when the insurance company's surplus has a negative value. Bankruptcy occurs if the total claim payment until time t exceeds the amount of initial capital plus earned premium until time t. The point of time where the bankruptcy occurs for the first time is defined as equation

$$T = \min\{t: t \geq 0 | U(t) < 0\}. \tag{3}$$

First-time ruin probability is defined as equation (4).

$$\psi(u) = P(T < \infty) = P(U(t) < 0); \quad t \geq 0 | U(0) = u). \tag{4}$$

Ruin probability in an unlimited time is expressed with equation (5).

$$\psi(u) = 1 - P(U(t) \geq 0) = 1 - \phi(u). \tag{5}$$

2.5 Adjustment Coefficient

Adjustment coefficient is the smallest positive solution of equation

$$M_X(r) = P1 + (1 + \theta)\mu r. \tag{6}$$

If the value of r is the smallest positive number, thus r value is referred to as adjustment coefficient [9].

2.6 Upper Limit of Ruin Probability

Theorem 1 [2].

If $\boldsymbol{U(t)}$ is the surplus process based on the compound Poisson claim accumulation process $\boldsymbol{S(t)}$, with $\boldsymbol{c > \lambda\mu}$, $\boldsymbol{\theta > 0}$, and $\boldsymbol{r > 0}$, then ruin probability $\boldsymbol{u \geq 0}$ is expressed as equation

$$\psi(u) = \frac{e^{-ru}}{E[e^{-rU(T)} | T < \infty]}. \tag{7}$$

Based on Theorem 1, ruin probability upper limit is as follows

$$\psi(u) \leq e^{-ru}, u \geq 0, r > 0, \tag{8}$$

with r being the adjustment coefficient. This inequation is referred to as Lundberg's inequality.

2.7 Premium Loading

Premium loading, notated as θ, is the additional premium percentage of the expected claim value. The greater the premium loading value is, the lesser the ruin probability would be. The amount of premium loading is equation

$$\theta = u\left(\frac{E[e^{-\frac{\ln\gamma}{u}X}] - 1}{-\mu\ln\gamma}\right) - 1, \tag{9}$$

where γ is maximum tolerance limit of the ruin probability.

2.8 Integro-differential Equation

Integro-differential equation is a type of mathematical equation that combines the derivative and integral of a function.

Theorem 2 [9].
The probability to survive a bankruptcy (survival) with no initial capital u is

$$\phi(0) = \frac{\theta}{1+\theta}. \tag{10}$$

Theorem 3 [9].
The probability to survive bankruptcy (survival) with initial capital $\boldsymbol{u}$ fulfills equation

$$\phi'(u) = \frac{\lambda}{c}\phi(u) - \frac{\lambda}{c}\int_0^u \phi(u-x)dF(x) = \frac{\lambda}{c}\phi(u) - \frac{\lambda}{c}\int_0^u \phi(u-x)f(x)dx. \tag{11}$$

2.9 Mixtures of Two Exponential Distributions

If random variable $\boldsymbol{X}$ is spread on two exponential distributions with parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ notated as $\boldsymbol{X \sim Exp(\alpha)}$ and $\boldsymbol{X \sim Exp(\beta)}$ and has a proportion $\boldsymbol{b}$ and $(\boldsymbol{1-b})$, then the probability density function of the mixtures of two exponential distributions is

$$f(x) = b\alpha e^{-\alpha x} + (1-b)\beta e^{-\beta x}, \tag{12}$$

where $\boldsymbol{x \geq 0, \alpha > 0, \beta > 0, 0 \leq b \leq 1}$ $[\boldsymbol{10}]$. The expected value of mixtures of two exponential distributions $\boldsymbol{X \sim Exp(\alpha)}$ and $\boldsymbol{X \sim Exp(\beta)}$ is

$$E[X] = \frac{b}{\alpha} + \frac{1-b}{\beta}. \tag{13}$$

2.10 Linear Differential Equation

The general form of homogeneous linear differential equation order-$\boldsymbol{n}$ is equation (14) by [11]:

$$L[z] = a_0 z^{(n)} + a_1 z^{n-1} + \cdots + a_{n-1}z' + a_n z = 0 \tag{14}$$

with $\boldsymbol{a_0 > 0}$. The characteristic equation of equation (14) is

$$P(p) = a_0 p^n + a_1 p^{n-1} + \cdots + a_{n-1}p + a_n = 0. \tag{15}$$

If the characteristic equation roots are real and different, then equation (15) has $\boldsymbol{n}$ different solutions $\boldsymbol{e^{p_1 u}, e^{p_2 u}, \ldots, e^{p_n u}}$. If these functions are linearly independent, then the general solution is

$$z(u) = A_1 e^{p_1 u} + \cdots + A_n e^{p_n u} \text{ [11]}. \tag{16}$$

3. RESULTS AND DISCUSSION

3.1 Model Assumption

The assumption used in this research was that the claim size followed the mixtures of two exponential distributions. The probability density function of the claim size was expressed by equation (12).

3.2 Model Derivation

Subtituting the probability density function $f(x)$ from claim distribution into the integro-differential equation resulted

$$\phi'(u) = \frac{\lambda}{c}\phi(u) - \frac{\lambda}{c}\int_0^u \phi(u-x)b\alpha e^{-\alpha x}dx - \frac{\lambda}{c}\int_0^u \phi(u-x)(1-b)\beta e^{-\beta x}dx. \tag{17}$$

For example, $y = u - x$, then equation (18) was obtained.

$$\phi'(u) = \frac{\lambda}{c}\phi(u) - \frac{\lambda}{c}b\alpha e^{-\alpha u}\int_0^u \phi(y)e^{\alpha y}dy - \frac{\lambda}{c}(1-b)\beta e^{-\beta u}\int_0^u \phi(y)e^{\beta y}dy. \quad (18)$$

3.3 Model Solution

To eliminate the integral, equation (18) was derived against $\boldsymbol{u}$ to obtain

$$\phi''(u) = \frac{\lambda}{c}\phi'(u) - \frac{\lambda}{c}b\alpha\phi(u) - \frac{\lambda}{c}(1-b)\beta\phi(u) + \frac{\lambda}{c}b\alpha^2\, e^{-\alpha u}\int_0^u \phi(y)e^{\alpha y}dy + \frac{\lambda}{c}(1-b)\beta^2\, e^{-\beta u}\int_0^u \phi(y)e^{\beta y}dy. \quad (19)$$

The two sides of equation (19) were multiplied with α,

$$\frac{\lambda}{c}b\alpha^2\, e^{-\alpha u}\int_0^u \phi(y)e^{\alpha y}dy = \frac{\lambda}{c}\alpha\phi(u) - \frac{\lambda}{c}(1-b)\alpha\beta e^{-\beta u}\int_0^u \phi(y)e^{\beta y}dy - \alpha\phi'(u). \quad (20)$$

Substituting equation (20) into equation (19) resulted

$$\phi''(u) = \frac{\lambda}{c}\phi'(u) - \frac{\lambda}{c}b\alpha\phi(u) - \frac{\lambda}{c}(1-b)\beta\phi(u) + \frac{\lambda}{c}\alpha\phi(u) - \alpha\phi'(u) - (\alpha-\beta)\left(\frac{\lambda}{c}(1-b)\beta e^{-\beta u}\int_0^u \phi(y)e^{\beta y}dy\right). \quad (21)$$

Then, equation (21) was derived against u to obtain equation

$$\phi'''(u) = \frac{\lambda}{c}\phi''(u) - \frac{\lambda}{c}b\alpha\phi'(u) - \frac{\lambda}{c}(1-b)\beta\phi'(u) + \frac{\lambda}{c}\alpha\phi'(u) - \alpha\phi''(u) - (\alpha-\beta)\left(\frac{\lambda}{c}(1-b)\beta\phi(u)\right) + (\alpha-\beta)\left(\frac{\lambda}{c}(1-b)\beta^2\, e^{-\beta u}\int_0^u \phi(y)e^{\beta y}dy\right). \quad (22)$$

The two sides of equation (22) were then multiplied with β

$$(\alpha-\beta)\left(\frac{\lambda}{c}(1-b)\beta^2 e^{-\beta u}\int_0^u \phi(y)e^{\beta y}dy\right) = \frac{\lambda}{c}\beta\phi'(u) - \frac{\lambda}{c}b\alpha\beta\phi(u) - \frac{\lambda}{c}(1-b)\beta^2\phi(u) + \frac{\lambda}{c}\alpha\beta\phi(u) - \alpha\beta\phi'(u) - \beta\phi''(u). \quad (23)$$

Substituting equation (23) into equation (22) resulted in equation

$$\phi'''(u) = \frac{\lambda}{c}\phi''(u) - \alpha\phi''(u) - \beta\phi''(u) - \frac{\lambda}{c}b\alpha\phi'(u) - \frac{\lambda}{c}\beta\phi'(u) + \frac{\lambda}{c}b\beta\phi'(u) + \frac{\lambda}{c}\alpha\phi'(u) + \frac{\lambda}{c}\beta\phi'(u) - \alpha\beta\phi'(u) - \frac{\lambda}{c}\alpha\beta\phi(u) + \frac{\lambda}{c}b\alpha\beta\phi(u) - \frac{\lambda}{c}b\alpha\beta\phi(u) + \frac{\lambda}{c}\alpha\beta\phi(u). \quad (24)$$

Finally, as equation (24) was eliminated, equation (25) was obtained.

$$\phi'''(u) + \left(-\frac{\lambda}{c} + \alpha + \beta\right)\phi''(u) + \left(\frac{\lambda}{c}b\alpha - \frac{\lambda}{c}b\beta - \frac{\lambda}{c}\alpha + \alpha\beta\right)\phi'(u) = 0. \quad (25)$$

Based on equation (2), the value of c was $c = (1+\theta)\lambda\mu$. Hence equation (26) was obtained

$$\frac{\lambda}{c} = \frac{\lambda}{(1+\theta)\lambda\mu} = \frac{\lambda}{(1+\theta)\lambda E[X]} = \frac{\alpha\beta}{(1+\theta)(b\beta+\alpha-b\alpha)}. \quad (26)$$

Substituting equation (26) into equation (25) resulted in equation (27).

$$\phi'''(u)+\left(\frac{-\alpha\beta+(\alpha+\beta)(1+\theta)(b\beta+\alpha-b\alpha)}{(1+\theta)(b\beta+\alpha-b\alpha)}\right)\phi''(u) + \left(\frac{b\alpha^2\beta-b\alpha\beta^2-\alpha^2\beta+(\alpha\beta)(1+\theta)(b\beta+\alpha-b\alpha)}{(1+\theta)(b\beta+\alpha-b\alpha)}\right)\phi'(u)=0. \quad (27$$

Furthermore, equation (27) could be expressed in equation (14) by taking $z=\phi'(u)$. It could be written as equation (28).

$$z''+a_1z'+a_2z=0. \quad (28)$$

The characteristic equation of equation (28) was

$$p^2+a_1p+a_2=0, \quad (29)$$

and its characteristic roots were obtained as follows (equation (30))

$$p_1=-\frac{1}{2}\left(\frac{-\alpha\beta+(\alpha+\beta)(1+\theta)(b\beta+\alpha-b\alpha)}{(1+\theta)(b\beta+\alpha-b\alpha)}\right) +\frac{1}{2}\sqrt{\frac{\left((-\alpha\beta)+(\alpha+\beta)(1+\theta)(b\beta+\alpha-b\alpha)\right)^2}{\left((1+\theta)(b\beta+\alpha-b\alpha)\right)^2}-\frac{4\left(b\alpha^2\beta-b\alpha\beta^2-\alpha^2\beta+(\alpha\beta)(1+\theta)(b\beta+\alpha-b\alpha)\right)}{(1+\theta)(b\beta+\alpha-b\alpha)}} \quad (30)$$

$$p_2=-\frac{1}{2}\left(\frac{-\alpha\beta+(\alpha+\beta)(1+\theta)(b\beta+\alpha-b\alpha)}{(1+\theta)(b\beta+\alpha-b\alpha)}\right) -\frac{1}{2}\sqrt{\frac{\left((-\alpha\beta)+(\alpha+\beta)(1+\theta)(b\beta+\alpha-b\alpha)\right)^2}{\left((1+\theta)(b\beta+\alpha-b\alpha)\right)^2}-\frac{4\left(b\alpha^2\beta-b\alpha\beta^2-\alpha^2\beta+(\alpha\beta)(1+\theta)(b\beta+\alpha-b\alpha)\right)}{(1+\theta)(b\beta+\alpha-b\alpha)}}. \quad (31)$$

Based on equation (16), the general solution of equation (27) was

$$\phi'(u)=A_1e^{p_1u}+A_2e^{p_2u}. \quad (32)$$

Moreover, by taking $u=0$ in equation (30), equation (33) was then obtained.

$$\phi'(0)=A_1+A_2 \quad (33)$$

Upon reviewing equations (10) and (8) for $u=0$, equation (34) was obtained

$$\phi'(0)=\frac{\lambda}{c}\phi(0)=\frac{\alpha\beta}{(1+\theta)(b\beta+\alpha-b\alpha)}\left(\frac{\theta}{1+\theta}\right)=\frac{\alpha\beta\theta}{(1+\theta)^2(b\beta+\alpha-b\alpha)}, \quad (34)$$

and thus equation (35) was also obtained.

$$A_1+A_2=\frac{\alpha\beta\theta}{(1+\theta)^2(b\beta+\alpha-b\alpha)}$$
$$A_2=\frac{\alpha\beta\theta}{(1+\theta)^2(b\beta+\alpha-b\alpha)}-A_1. \quad (35)$$

Then, integrating equation (32) against u resulted

$$\phi(u)=\frac{A_1}{p_1}e^{p_1u}+\frac{A_2}{p_2}e^{p_2u}+A_{3,} \quad (36)$$

Since $\phi(\infty)=1$, then $A_3=1$, and

$$\phi(u) = \frac{A_1}{p_1} e^{p_1 u} + \frac{A_2}{p_2} e^{p_2 u} + 1. \tag{37}$$

By taking $u = 0$, the following equation was

$$\phi(0) = \frac{A_1}{p_1} + \frac{A_2}{p_2} + 1.$$
$$\frac{\theta}{1+\theta} = \frac{A_1}{p_1} + \frac{A_2}{p_2} + 1$$
$$\frac{p_1 p_2}{1+\theta} = -A_1 p_2 - A_2 p_1,$$

Substituting equation (35) resulted

$$\frac{p_1 p_2}{1+\theta} = -p_2 A_1 - p_1 \left(\frac{\alpha\beta\theta}{(1+\theta)^2 (b\beta + \alpha - b\alpha)} - A_1 \right)$$
$$\Leftrightarrow A_1 = \frac{1}{(p_1 - p_2)} \left(\frac{p_1 p_2}{1+\theta} + \frac{\alpha\beta\theta p_1}{(1+\theta)^2 (b\beta + \alpha - b\alpha)} \right). \tag{38}$$

Furthermore, substituting equation (38) into equation (35) resulted

$$A_2 = \frac{\alpha\beta\theta}{(1+\theta)^2 (b\beta + \alpha - b\alpha)} - \frac{1}{(p_1 - p_2)} \left(\frac{p_1 p_2}{1+\theta} + \frac{\alpha\beta\theta p_1}{(1+\theta)^2 (b\beta + \alpha - b\alpha)} \right). \tag{39}$$

The ruin probability model could be formulated by substituting equation (37) into equation (5) which then resulted

$$\psi(u) = 1 - \phi(u)$$
$$\psi(u) = -\frac{A_1}{p_1} e^{p_1 u} - \frac{A_2}{p_2} e^{p_2 u}. \tag{40}$$

where

$$A_1 = \frac{1}{(p_1 - p_2)} \left(\frac{p_1 p_2}{1+\theta} + \frac{\alpha\beta\theta p_1}{(1+\theta)^2 (b\beta + \alpha - b\alpha)} \right)$$

$$A_2 = \frac{\alpha\beta\theta}{(1+\theta)^2 (b\beta + \alpha - b\alpha)} - \frac{1}{(p_1 - p_2)} \left(\frac{p_1 p_2}{1+\theta} + \frac{\alpha\beta\theta p_1}{(1+\theta)^2 (b\beta + \alpha - b\alpha)} \right).$$

$$p_1 = -\frac{1}{2} \left(\frac{-\alpha\beta + (\alpha+\beta)(1+\theta)(b\beta + \alpha - b\alpha)}{(1+\theta)(b\beta + \alpha - b\alpha)} \right) + \frac{1}{2} \sqrt{ \frac{\left((-\alpha\beta) + (\alpha+\beta)(1+\theta)(b\beta+\alpha-b\alpha)\right)^2}{\left((1+\theta)(b\beta+\alpha-b\alpha)\right)^2} - \frac{4\left(b\alpha^2\beta - b\alpha\beta^2 - \alpha^2\beta + (\alpha\beta)(1+\theta)(b\beta+\alpha-b\alpha)\right)}{(1+\theta)(b\beta+\alpha-b\alpha)} },$$

$$p_2 = -\frac{1}{2} \left(\frac{-\alpha\beta + (\alpha+\beta)(1+\theta)(b\beta + \alpha - b\alpha)}{(1+\theta)(b\beta + \alpha - b\alpha)} \right) - \frac{1}{2} \sqrt{ \frac{\left((-\alpha\beta) + (\alpha+\beta)(1+\theta)(b\beta+\alpha-b\alpha)\right)^2}{\left((1+\theta)(b\beta+\alpha-b\alpha)\right)^2} - \frac{4\left(b\alpha^2\beta - b\alpha\beta^2 - \alpha^2\beta + (\alpha\beta)(1+\theta)(b\beta+\alpha-b\alpha)\right)}{(1+\theta)(b\beta+\alpha-b\alpha)} }.$$

3.4 Model Analysis

Based on the obtained ruin probability model in equation (40), it could be analyzed that ruin probability was affected by some factors, namely expected claim value, additional premium percentage or premium loading ($\boldsymbol{\theta}$), and initial capital $\boldsymbol{u}$.

The greater the expected claim value is, the lesser the parameter values $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ would be. On the contrary, the lesser the expected claim value is, the greater the parameter values $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ would be. When the expected claim value gets greater, the values in $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ will be negative and get greater as well (closer to zero), where $\boldsymbol{p_2} < \boldsymbol{p_1}$. This was because the values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ were greater in their numerators. These $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ values did not affect both $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ values. In equations $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$, if $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ were negative and closer to zero, then $\boldsymbol{A_1}$ value would be lesser and $\boldsymbol{A_2}$ would be greater. Based on these values, the ruin probability would be greater. Therefore, the greater the expected claim value is, the greater the ruin probability $\boldsymbol{\psi}(\boldsymbol{u})$ would be.

When the premium loading ($\boldsymbol{\theta}$) was greater, then the values of $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ would be negative and lesser (more negative) with $\boldsymbol{p_2} < \boldsymbol{p_1}$. This was because in equations $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$, $\boldsymbol{\theta}$ was greater in the denominator. These $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ values affected the values of $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$. In equations $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$, the premium loading ($\boldsymbol{\theta}$) was greater in the denominator, making the values of $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ positive and lesser. Therefore, the ruin probability would be lesser. This showed that the greater the premium loading ($\boldsymbol{\theta}$) is, the lesser the ruin probability $\boldsymbol{\psi}(\boldsymbol{u})$ would be.

When the initial capital $\boldsymbol{u}$ was greater with $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ being negative and lesser (more negative), then the value of $\boldsymbol{e^{p_1 u}}$ would be positive and closer to zero. This made the ruin probability lesser. Therefore, the greater the initial capital $\boldsymbol{u}$ is, the lesser the ruin probability $\boldsymbol{\psi}(\boldsymbol{u})$ would be. Based on this analysis, it could be concluded that:

1. The greater the expected claim value is, the greater the ruin probability $\psi(u)$ would be.
2. The greater the additional premium percentage or premium loading (θ) is, the lesser the ruin probability $\psi(u)$ would be.
3. The greater the initial capital u is, the lesser the ruin probability $\psi(u)$ would be.

3.5 Model Application

The ruin probability $\psi(u)$ model was applied using the Python programming language for its numeric calculation. This application was divided into three cases. In the first case, the changes in parameter α values were analyzed, namely, 0.5, 1, and 2, while other parameters remained the same, namely $b = 0.5$, $\beta = 0.5$, dan $\theta = 0.1$. In Case 2, the changes in parameter β values were analyzed, namely 0.5, 1, and 2, while other parameters remained the same, namely $b = 0.5$, $\alpha = 2$, and $\theta = 0.1$. Furthermore, in the third case, the changes in parameter θ values were analyzed, namely 0.1, 0.2, and 0.3, while other parameters remained the same, namely $b = 0.5$, $\alpha = 2$, and $\beta = 0.5$. In each case, the values of initial capital u used were $0, 1, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$. The results of ruin probability calculation for each case are presented in Figure 1.

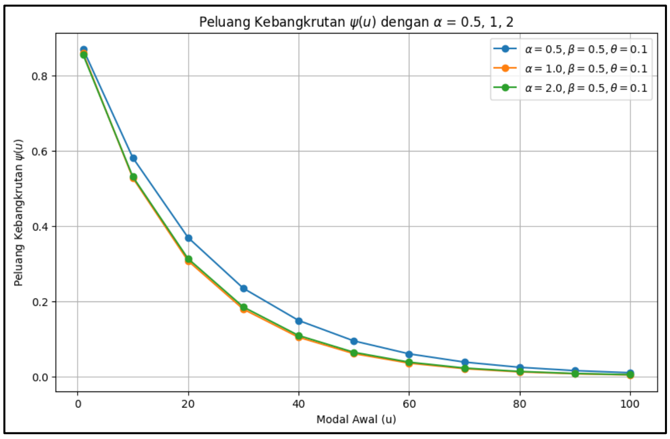

Figure 1. Chart of Ruin Probability $\psi(u)$ against Initial Capital u when α Changes and b, β, θ Remain the Same.

Based on Figure 1, the ruin probability decreased significantly as the initial capital u increased. In addition, the greater parameter α value, indicating a decreased expected claim value, resulted in lesser ruin probability for every initial capital u value. When the initial capital $u = 0$ for every α value, the ruin probability was the same. At a large initial capital, i.e., when $u = 100$, for every α, the ruin probability was so small and did not differ significantly. This showed that when the company's initial capital was so large, claims had no significant effect.

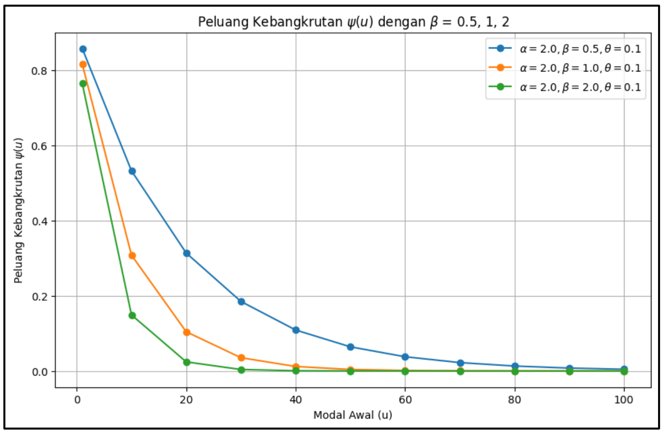

Figure 2. Chart of Ruin Probability $\psi(u)$ Against Initial Capital u where β Changes and b, α, θ Remain the Same

Based on Figure 2, the ruin probability decreased significantly as the initial capital u increased. In addition, the greater the parameter β value, indicating a decreased expected claim value, made the ruin probability decrease for every initial capital u value. Compared to Case 1, the ruin probability in Case 2 decreased more significantly. This was because of the lesser expected claim value in Case 2. When the initial capital $u = 0$ for every β value, the ruin probability was the same. At a large initial capital $u = 100$, the ruin probability for every β value, was so small.

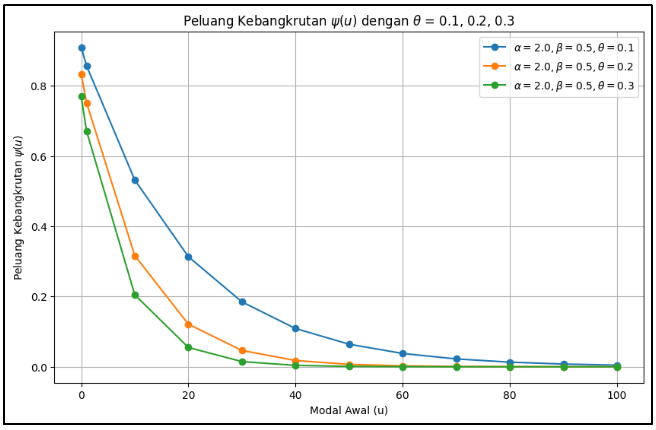

Figure 3. Chart of Ruin Probability $\psi(u)$ Against Initial capital u where θ Changes and b, α, β Remain the Same

Based on Figure 3, it could be seen that the bankruptcy probability $\psi(u)$ decreased significantly as the initial capital u increased. Additionally, the greater premium loading (θ) value affected the ruin probability decrease significantly. At initial capital $u = 0$, the ruin probability decreased significantly as the premium loading (θ) increased. In this case, the ruin probability decreased faster than in Case 1. This showed that companies with great premium had better survival against bankruptcy.

Based on the three cases, it could be concluded that:

1. The Effect of Initial Capital $\boldsymbol{u}$
   The ruin probability $\psi(u)$ decreased significantly as the initial capital u increased. This showed that a company with high initial capital had stronger survival ability.
2. The Effect of Premium Rate
   The greater premium loading (θ) had significant effect on ruin probability decrease. This indicated that higher premium rate could effectively reduce the ruin probability.
3. The Effect of Claim
   The amount of expected claim value was proportional to the ruin probability $\psi(u)$. The greater the expected claim value is (α and β are lesser), the greater the ruin probability would be. This showed that the amount of claim significantly affected the ruin probability.

4. CONCLUSIONS

The conclusions of this study are

1. The integro-differential approach is quite effective and flexible for use in modeling ruin probability. The integro-differential approach for the case of claims is exponentially distributed successfully provides a general solution for the ruin probability model which includes the parameters u, b, α, β, θ with the initial capital u ≥ 0. Theoretically based on the general solution obtained, the analysis of these parameters provides the following information.
   a. If the expected value of the large claim distribution is greater, then the probability of ruin ψ(u) is greater. This happens if the values of parameter α, β become smaller.
   b. If the premium loading (θ) is greater, then the ruin probability ψ(u) is smaller
   c. If the initial capital u is increased, then the ruin probability ψ(u) will be smaller.
2. The research results showed that ruin probability ψ(u) was inversely proportional to initial capital u and premium loading (θ), and the ruin probability ψ(u) was proportional to the amount of expected claim value. In other words, the ruin probability ψ(u) decreased significantly as the initial capital u

increased, the premium loading (θ) increased and the expected claim value decreased. Therefore, the greater the insurance company's surplus, the lesser the ruin probability would be.

3. The simulations conducted provide the following practical insights.
   a. For Case 1, when the company's initial capital is very large, claims do not have a significant effect on the chance of bankruptcy.
   b. Compared to Case 1, the probability of ruin in Case 2 decreases significantly. This is due to the smaller expected value of claims in Case 2.
   c. For Case 3, a higher premium loading (θ) significantly reduces the ruin probability. This indicates that a higher premium rate can effectively reduce ruin probability.
4. The implication of this research is to demonstrates the effectiveness and flexibility of the integro-differential approach. This approach can be used to model the probability of ruin in cases involving mixtures of three or more exponential distributions, and other claim amount distributions, such as the normal, gamma, Rayleigh, and other exponential distribution families.

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